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14/ENG01/019
CHEMICAL ENGINEERING

Assignment IV

1. $$y(s) = \frac{G_p(s) G_f(s) G_c(s)}{1 + G_p(s) G_f(s) G_c(s) G_m(s)} \cdot y_{sp}(s)$$

For a P only controller, $G_c = k_c$

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_p(s) G_f(s) G_c(s)}{1 + G_p(s) G_f(s) G_c(s) G_m(s)}$$

$$1 + G_p(s) G_f(s) G_c(s) G_m(s) = 0$$ [characteristic equation]

From figure, 1:

i. $$G_p(s) = \frac{1}{\tau_p s + 1}$$

and $\tau_p = 1$

$$G_p(s) = \frac{1}{s+1}$$

ii. $$G_f(s) = \frac{1}{\tau_f s + 1}$$

and $\tau_f = 1/2$

$$G_f(s) = \frac{1}{1/2\, s + 1}$$

$\tau_I \quad \tau_D$ $\quad G_C = \frac{K_C}{\tau_I s}$

$G_F = K_P$

$$G_{f(s)} = \frac{\frac{1}{1/2}}{\frac{1/2}{1/2} s + \frac{1}{1/2}}$$

$$G_{f(s)} = \frac{2}{s + 2}$$

iii. $G_C(s) = K_C(s)$

iv. $G_m(s) = \frac{1}{\tau_m s + 1}$

$$\tau_m = 1/3$$

$$G_m(s) = \frac{1}{1/3\, s + 1}$$

$$G_m(s) = \frac{3}{s + 3}$$

Substituting ~~the~~ ~~values~~ & $G_p(s)$, $G_f(s)$, $G_c(s)$ and $G_m(s)$ into the characteristic equation, we have:

$$1 + \frac{1}{s+1} \cdot \frac{2}{s+2} \cdot K_c \cdot \frac{3}{s+3} = 0$$

$$1 + \frac{6K_c}{(s+1)(s+2)(s+3)} = 0$$

$$\frac{6\cancel{k}c}{(s\cancel{+1})(s\cancel{+2})(s\cancel{+3})} = \cancel{-1}$$

$$\frac{1}{1} + \frac{6kc}{(s+1)(s+2)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s+3)+6kc}{(1)(s+1)(s+2)(s+3)} \cancel{=} 0$$

$$(s+1)(s+2)(s+3)+6kc = 0$$

$$(s^2+2s+s+2)(s+3)+6kc = 0$$

$$(s^2+3s+2)(s+3)+6kc = 0$$

$$s(s^2+3s+2)+3(s^2+3s+2)+6kc = 0$$

$$s^3+3s^2+2s+3s^2+9s+6+6kc = 0$$

$$s^3+6s^2+11s+6+6kc = 0$$

Using Routh Criterium, we have the general formula to be:

Row 1:	a_0	a_2	a_4	a_6	---
Row 2:	a_1	a_3	a_5	a_7	---
Row 3:	A_1	A_2	A_3	---	
Row 4:	B_1	B_2	B_3	---	
Row 5:	C_1	C_2	C_3	---	

~~Substitution~~ (constants)

and $a_0 = 1$, $a_1 = 6$, $a_2 = 11$, $a_3 = 6 + 6K_c$

Substituting the coefficients into the Routh criterium, we have:

Row 1: 1 11

Row 2: 6 $6 + 6K_c$

Row 3: $10 - K_c$ 0

Row 4: $6 + 6K_c$ 0

$$A_1 = \frac{(6 \times 11) - (1 \times (6 + 6K_c))}{6}$$

$$= \frac{66 - (6 + 6K_c)}{6}$$

$$= \frac{66 - 6 - 6K_c}{6}$$

$$= \frac{60 - 6K_c}{6}$$

$$A_1 = 10 - K_c$$

$A_2 = 0$, $A_3 = 0$

$$B_1 = \frac{(10-k_c)(6+6k_c) - (6)(0)}{10-k_c}$$

$$= \frac{60 + 60k_c - 6k_c - 6k_c^2 - 0}{10-k_c}$$

$$= \frac{-6k_c^2 + 54k_c + 60}{10-k_c}$$

$$= \frac{(k_c-10)(k_c+1)}{10-k_c}$$

$$= \frac{-\cancel{(-k_c+10)}(-k_c \mp 1)}{\cancel{10-k_c}}$$

$$= -(-k_c \mp 1)$$

$$= \cancel{-k_c} + k_c + 1$$

$$=$$

$$B_1 = \frac{\cancel{(10-k_c)}(6+6k_c)}{\cancel{10-k_c}}$$

$$B_1 = 6+6k_c$$

$B_2 = 0$, $C_1 = 0$ .....

For the control system to be stable, using Routh's criterium, all the values in the 1st column must be positive (greater than 0).

$\therefore$ From Row 3,

$$10 - K_c > 0$$

$$-K_c > -10$$

$$K_c < \frac{-10}{-1}$$

$$K_c < 10$$

From Row 4,

$$6 + 6K_c > 0$$

$$6K_c > -6$$

$$K_c > \frac{-6}{6}$$

$$K_c > -1$$

$\therefore K_c < 10$ and $K_c > -1$

$\therefore$ The ~~range~~ range of values of K_c ~~for~~ for which the system is stable is $-1 < K_c < 10$

$\therefore K_c = [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9]$

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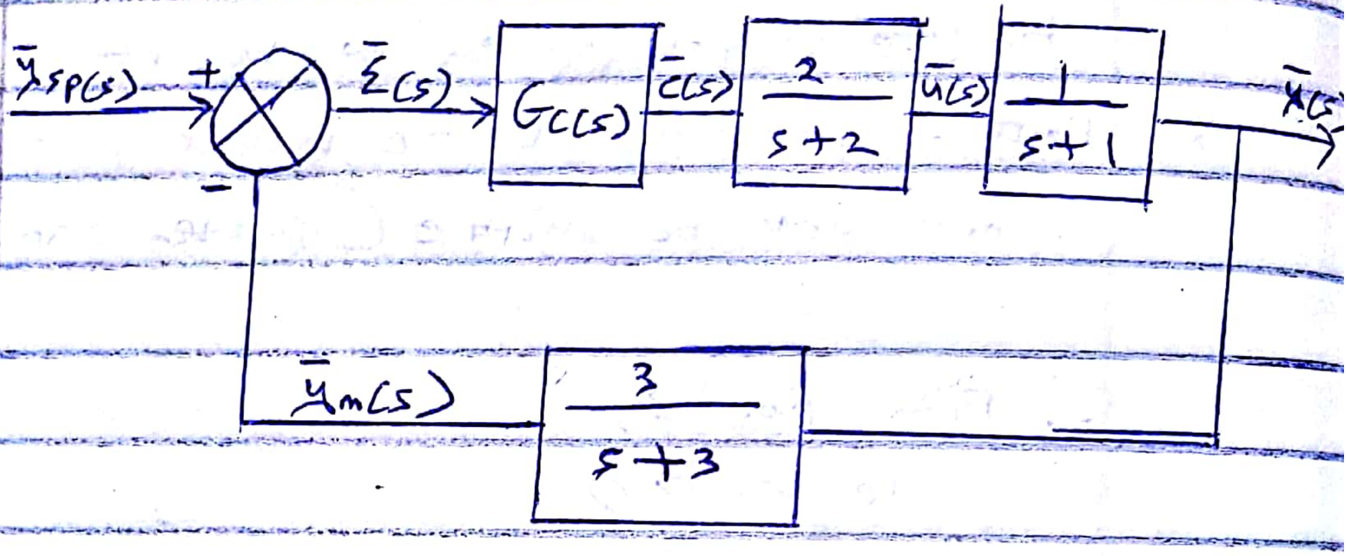

$$y(s) = \frac{G_p(s) G_f(s) G_c(s)}{1 + G_p(s) G_f(s) G_c(s) G_m(s)} \cdot y_{sp}(s)$$

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_p(s) G_f(s) G_c(s)}{1 + G_p(s) G_f(s) G_c(s) G_m(s)}$$

$$1 + G_p(s) G_f(s) G_c(s) G_m(s) = 0$$ [characteristic equation]

For a PI controller, $G_c = K_c + \frac{K_c}{\tau_I s}$

$$K_c = 5, \quad \tau_I = 0.25$$

$$G_c = \frac{5}{1} + \frac{5}{0.25s}$$

$$G_c = \frac{1.25s + 5}{0.25s}$$

$$G_c = \frac{\frac{1.25s}{0.25} + \frac{5}{0.25}}{\frac{0.25s}{0.25}}$$

$$G_c = \frac{5s + 20}{s}.$$

Substituting G_p, G_f, G_c and G_m into the characteristic equation, we have:

$$1 + \frac{1}{s+1} \cdot \frac{2}{s+2} \cdot \frac{5s+20}{s} \cdot \frac{3}{s+3} = 0$$

$$1 + \frac{6(5s+20)}{(s+1)(s+2)(s)(s+3)} = 0$$

$$\frac{1}{1} + \frac{30s + 120}{(s+1)(s+2)(s)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s)(s+3) + 30s + 120}{(s+1)(s+2)(s)(s+3)} = 0$$

$$(s+1)(s+2)(s)(s+3) + 30s + 120 = 0$$

$$(s)(s+1)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^2 + s)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^3 + 2s^2 + \cancel{5}s^2 + \overset{2s}{\cancel{10}})(s+3) + 30s + 120 = 0$$

$$s^4 + 2s^3 + s^3 + 2s^2 + 3s^3 + 6s^2 + 3s^2 + 6s + 30s + 120 = 0$$

$$s^4 + 6s^3 + 11s^2 + 36s + 120 = 0$$

Using Routh criterion, we have

Row 1: 1 11 120

Row 2: 6 36 0

Row 3: 5 120 0

Row 4: -108 0

Row 5: 120

$$A_1 = \frac{(6)(11) - (1)(36)}{6}$$

$$= \frac{66 - 36}{6}$$

$$= 5$$

$$A_2 = \frac{(6)(120) - (1)(0)}{6}$$

$$= \frac{720 - 0}{6}$$

$$A_2 = 120$$

$$A_3 = 0$$

$$B_1 = \frac{(5)(36) - (6)(120)}{5}$$

$$= \frac{180 - 720}{5}$$

$$= -108 \qquad B_2 = 0, \; B_3 = 0$$

$$C_1 = \frac{(-108)(120) - (5)(0)}{-108}$$

$$= 120$$

$$C_2 = 0 \quad , C_3 = 0$$

The system is said to be unstable because not ~~not~~ all the values in the first column of the Routh criterium is positive.

For the system to be stable, all the values in the first column of the criterium must be positive.