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ENG 381
Assignment IV
H/CoeCom/06

① $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2x y^{(1)} + 2y = 0$

let $v = 1-x^2$ $\quad u = y^{(2)}$

$v' = -2x$ $\quad u^{(n)} = y^{(n+2)}$

$v'' = -2$ $\quad u^{(n-1)} = y^{(n+1)}$

$u^{(n-2)} = y^{(n)}$

$$y^{(n)} = y^{(n+2)}(1-x^2) + n y^{(n+1)}(-2x) + \frac{n(n-1)}{2!} y^{n}(-2)$$

$y^{(n)} =$

$-2x y^{(1)}$

$v = -2x$ $\quad u = y^{(1)}$

$v' = -2$ $\quad u^{(n)} = y^{(n+1)}$

$u^{(n-1)} = y^{(n)}$

$y^{(n+1)}(-2x) + n y^{(n)}(-2)$

For $2y$

$v = 2$ $\quad u = y$

$u^{(n)} = y^{(n)}$

$= y^{(n)}(2)$

$$y^{(n)} = y^{(n+2)}(1-x^2) + n y^{(n+1)}(-2x) + \frac{n(n-1)}{2!} y^{n}(-2) + y^{(n+1)}(-2x) + n y^{(n)}(-2) + y^{(n)}(2)$$

at $x = 0$

$$y^{(n)} = y^{(n+2)}(1) + n y^{(n+1)}(0) + \frac{n(n-1)}{2!} y^{n}(-2) + y^{(n+1)}(0) + n y^{(n)}(-2) + y^{(n)}(2)$$

$$y^{(n)} = y^{(n+2)} - \frac{2\, n(n-1)}{2} y^{n} + n y^{(n)}(-2) + 2y^{(n)}$$

$$y^{(n)} = y^{(n+2)} - n(n-1)y^{n} + n y^{(n)}(-2) + 2y^{(n)}$$

$$y^{(n)} = y^{(n+2)} + y^{(n)}(1-n^2-2n+2)$$

$$y^{(n)} = y^{(n+2)} + y^{(n)}(-n^2-2n+3)$$

$y^{(n+2)} + y^{(n)}(-n^2-2n+3)$

$y^{(n+2)} = -y^{(n)}(-n^2-2n+3)$

$y^{(n+2)} = y^{(n)}(n^2+2n-3)$

$y^{(n)} = y^{(2)}(-3) = -3y^{(0)}$

when $n=1$

$y^{(3)} = y^{(1)}(1^2+2-3)$

$y^{(3)} = 0$

when $n=2$

$y^{(4)} = y^{(2)}(4+4-3)$

$y^{(4)} = 5y^{(2)} = (5)(-3)y^{(0)}$

when $n=3$

$y^{(5)} = y^{(3)}(9+6-3)$

$y^{(5)} = y^{(3)}(12) = 0$

when $n=4$

$y^{(6)} = y^{(4)}(16+8-3)$

$y^{(6)} = y^{(4)}(21) = (21)(5)(-3)y^{(0)}$

when $n=5$

$y^{(7)} = y^{(5)}(25+10-3)$

$y^{(7)} = y^{(5)}(32) = 0$

when $n=6$

$y^{(8)} = y^{(6)}(36+12-3)$

$y^{(8)} = y^{(6)}(45) = (45)(21)(5)(-3)y^{(0)}$

Maclaurin Series

$$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{(4)})_0$$

$$y = y_0 + xy' + \frac{x^2}{2!}(-3y^{(0)}) + \frac{x^4}{4!}(5)(-3)y^{(0)} + \frac{x^6}{6!}(21)(5)(-3)y^{(0)} + \frac{x^8}{8!}(45)(21)(5)(-3)y^{(0)}$$

$$y = y_0 + xy' + \frac{x^2}{2}(-3y^{0}) + \frac{x^4}{8}(-5)y^{0} + \frac{x^6}{16}(-7y^{0}) + \frac{x^8}{128}(-45y^{0}) + \dots$$

②(i) $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

(ii) $\sin 4t + \cos 4t$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

(iii) $t^2 + 2t^3 - t + 4$

$\frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t}\cos 5t$

$= \frac{(s+2)}{(s+2)^2+5^2}$

$= \frac{(s+2)}{(s+2)^2+25}$

(v) $t\sin 3t$

$L(\sin 3t) = \frac{3}{s^2+3^2}$

$L[t\sin 3t] = \frac{-d}{ds}(f(s))$

$= \frac{-d}{ds}\left(\frac{3}{s^2+3^2}\right)$

$$\frac{-4s(s^2+1) - (-2s^2)(2s)}{(s^2+1)^2}$$

$$= \frac{\cancel{-10(s^2+1) + 20s^2}}{\cancel{(s^2+1)^3}}$$

$$\frac{-4s(s^2+1) + 4s^3}{(s^2+1)^2}$$

$$= \frac{\cancel{-10s^2 - 10 + 20s^2}}{\cancel{(s^2+1)^3}}$$

$$= \frac{\cancel{-4s^3} - 4s + \cancel{4s^3}}{(s^2+1)^2}$$

$$= \frac{10s^2 - 10}{(s^2+1)^3}$$

$$= \frac{-4s}{(s^2+1)^2}$$

(xii) $\frac{\sinh 2t}{t}$

$$L\{\sinh 2t\} = \frac{2}{s^2-4}$$

$$L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2-4}\, d\sigma$$

$$= 2\int_{\sigma=s}^{\infty} \frac{1}{\sigma^2-4}\, d\sigma$$

$$\cancel{u = \sigma^2 - 4}$$

$$\cancel{\frac{du}{d\sigma} = 2\sigma}$$

$$= 2\int_{\sigma=s}^{\infty} \frac{-1}{-1}\, \frac{1}{\sigma^2-4}\, d\sigma$$

$$= -2\int_{\sigma=s}^{\infty} \frac{1}{2^2-\sigma^2}\, d\sigma$$

$$= -2\left(\frac{1}{2}\cosh^{-1}\frac{\sigma}{2}\right)_s^{\infty}$$

$$= \frac{(s^2+s^2)(1) - (s)(2s)}{(s^2+4)^2}$$

$$= \frac{-6s}{(s^2+1)^2}$$

$$= -\left(\frac{-6s}{(s^2+4)^2}\right)$$

$$= \frac{6s}{(s^2+4)^2}$$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

$$L\{e^{-t} - e^{-2t}\}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty}\left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right) d\sigma$$

$$= \Big(\ln(\sigma+1) - \ln(\sigma+2)\Big)\Big|_{\sigma=s}^{\infty}$$

$$= \ln\left(\frac{\sigma+1}{\sigma+2}\right)\Big|_{\sigma=s}^{\infty}$$

$$\ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\left(\frac{s+1}{s+2}\right)$$

$$= \ln(1) - \ln\left(\frac{s+1}{s+2}\right)$$

$$= -\ln\left(\frac{s+1}{s+2}\right)$$

$$= \ln\left(\frac{s+1}{s+2}\right)^{-1}$$

$$= \ln\left(\frac{s+2}{s+1}\right)$$

(viii) $e^{4t}\cos 2t$

$$\frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$= \frac{(s-4)}{(s-4)^2+4}$$

(ix) $t^3 + 4t^2 + 5$

$$= \frac{6}{s^4} + \frac{4 \cdot 2}{s^3} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{6}{s^3} + \frac{5}{s}$$

(x) $e^{3t}(t^2+4)$

$$L\{t^2+4\} = \frac{2}{s^3} + \frac{4}{s}$$

$$\therefore \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

(xi) $t^2\cos t$

$$L\{\cos t\} = \frac{s}{s^2+1}$$

$$L\{t^2\cos t\} = \frac{d}{ds}\left(-\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right)$$

$$L\{t\cos t\} = \frac{(\cancel{s^2}+1)(1) - (s)(2s)}{(s^2+1)^{\cancel{2}}}$$

$$= \frac{\cancel{-4s^3} - 2s^2}{(s^2+1)}$$

$$L(t^2\cos t) = \frac{(s^2+1)(-4s) - (-2s^2)(2s)}{(s^2+1)^{2}}$$

$$= \frac{(-4s)(s^2+1) + (-2s^2)(2s)}{(s^2+1)^{2}}$$

$A+B=1$

$-2A-4B-\frac{1}{2}=-3$

$-2A-4B=-3+\frac{1}{2}$

$-2A-4B=-\frac{5}{2}$

$A+B=1 \qquad A=1-B$

$2\cancel{A+4B}=\frac{5}{2}$

$2(1-B)+4B=\frac{5}{2}$

$2-2B+4B=\frac{5}{2}$

$2B=\frac{5}{2}-2$

$2B=\frac{1}{2}$

$B=\frac{1}{4}$

$A+\frac{1}{4}=1$

$A=\frac{3}{4}$

$\therefore \frac{3/4}{s-3}+\frac{1/4}{(s-1)}-\frac{1/2}{(s-1)^2}$

$f(t)=\frac{3}{4}e^{3t}+\frac{1}{4}e^{t}-\frac{1}{2}te^{t}$

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(v) $\frac{s-5}{s^2+4s+20}=\frac{Bs+C}{s^2+4s+20}$

$\frac{s-5}{s^2+4s+20}=\frac{Bs+C}{s^2+4s+20}$

$s-5=Bs+C$

$B=1$

$C=-5$

$\frac{s-5}{s^2+4s+20}$

$s+2 \qquad \frac{s-5}{(s+2)^2+16}$

$=\frac{s}{(s+2)^2+16}-\frac{5}{(s+2)^2+16}$

$=\frac{s}{\ldots}-\frac{5}{(\ldots)^2+4^2}$

(i)

$\frac{s-5}{(s-3)(s-4)}=\frac{A}{s-3}+\frac{B}{s-4}$

$s-5=A(s-4)+B(s-3)$

when $s=4$

$4-5=B(4-3)$

$-1=B$

$B=-1$

when $s=3$

$3-5=A(3-4)$

$-2=A(-1)$

$A=2$

$F(s)=\frac{2}{s-3}-\frac{1}{s-4}$

$f(t)=2e^{3t}-e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)}=\frac{A}{(s-2)}+\frac{B}{(s-4)}$

$2s-6=A(s-4)+B(s-2)$

at $s=4$

$8-6=B(4-2)$

$2=2B$

$B=1$

at $s=2$

$4-6=A(2-4)$

$-2=-2A$

$A=1$

$F(s)=\frac{1}{s-2}+\frac{2}{s-4}$

$f(t)=e^{2t}+e^{4t}$

(iii) $\frac{5s-8}{s(s-4)}=\frac{A}{s}+\frac{B}{s-4}$

$5s-8=A(s-4)+Bs$

$5s-8=As-4A+Bs$

$A+B=5$

$-4A=-8$

$A=2$

$2+B=5$

$B=5-2$

$B=3$

$\frac{2}{s}+\frac{3}{s-4}$

$2+3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{(s-4)(s+1)}{(s-3)(s-1)^2}=\frac{A}{(s-3)}+\frac{B}{(s-1)}+\frac{C}{(s-1)^2}$

$(s-4)(s+1)=A(s-1)^2+B(s-1)(s-3)+C(s-3)$

$(s-4)(s+1)=A(s^2-2s+1)+B(s^2-4s+3)+C(s-3)$

$(s^2-3s-4)=As^2-2As+A+Bs^2-4Bs+3B+Cs-3C$

$A+B=1$

$-2A-4B+C=-3$

$A+3B-3C=1$

$A=1-B$

$-2(1-B)-4B+C=-3 \quad = \quad -2+2B-4B+C=-3$

$1-B+3B-3C=1 \quad\quad 1+2B-3C=1$

$-2B+C=-1$

$+\quad 2B-3C=0$

$0\ \ -2C=1$

$C=-\frac{1}{2}$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7 \times \frac{4}{4}}{(s+2)^2+4^2}$$

$$\boxed{e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t}$$

$$\frac{s-5}{(s+2)^2+16}$$

$$\frac{s+2-2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$A+B=1$

$2A+4B=5/2$

$A=1-B$

$2(1-B)+4B=5/2$

$2-2B+4B=5/2$

$2B=5/2-2$

$A+B=1$

$-2A-4B-1/2=-3$

$-2A-4B=-3+1/2$

$-2A-4B=-5/2$