ENG 381 Assignment 4

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Civil Engineering

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W_1 = (1-x^2)y''$ $\quad v = (1-x^2),\ v' = 2x,\ v'' = 2$

$U = y'',\ U = y^{(n+2)}$

$y^n = U^n v + nU^{(n-1)}v' + \frac{n(n-1)U^{(n-2)}v''}{2}$

$= y^{(n+2)}(1-x^2) + n(y^{(n+1)})-2x + \frac{n(n-1)y^{n-2}}{2!}$

$= (1-x^2)y^{n+2} - 2xny^{n+1} + (n^2+n)y^n$

$W^2 = 2xy'$

$U = y',\ v = -2x$

$U^n = y^{(n+1)}\quad v' = 2$

$= U^n v + nU^{n-1}v'$

$= y^{(n+1)}(-2x) + ny^n(-2)$

$= -2xy^{n+1} - 2ny^n$

$W_3 = 2y,\ U = y,\ U^n = y^n,\ v = 2,\ v' = 0$

$= 2y^n$

$(1-x^2)y^{n+2} - 2xny^{(n+1)} - (n-n^2)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

$(1-x^2)y^{n+2} - 2xy^{n+1}(n+1) + y^n(-n^2-n+2) = 0$

at $x = 0$

$(1-0)y^{n+2} + y^n(-n^2-n+2) = 0$

$(y^{n+2})_0 = -[y^n]_0[n^2-n+2] = 0$

$[y^{n+2}]_0 = [y^n]_0[n^2-n+2]$

at $n=0$, $[y^2]_0 = -y[y^0]_0 - (0-0+2)$

$= y_0 = -2y^0$

at $n=1$, $[y^3]_0 = 0$

at $n=2$, $[y^4]_0 = 4[-2][y^0]_0$

at $n=3$ $[y^3]_0 = 0[10] = 0$

at $n=4$, $[y^6]_0 = (18)(4)[-2][y^0]_0$

at $n=6$, $[y^7]_0 = 0$

$y^n = [y]_0 + x[y']_0 - 2x^2[y^0] - \frac{x^4}{3!}[y]^0 - \frac{x^6}{5}[y^5]$

$y^n = [y]_0\left(1 - 2x^2 - \frac{x^4}{3} - \frac{x^6}{5} + \cdots\right) + x[y']_0$

2) i) $L(3e^{-4t} - 5e^{4t}) = 3\times L(e^{-4t}) - 5\times L(e^{4t})$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii) $L(t^3 + 2t^2 - t + 4$

$= L[t^3] + 2L[t^2] - L[t] - L[4]$

$= \frac{3}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+5^2}$

$= \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+25}$

v) $L\left[\frac{t\sin 3t}{t}\right] = \frac{3}{s^2+9}$

$L[t\sin 3t] = \frac{-\delta}{\delta x}\left[\frac{3}{s^2+9}\right]$

using quotient rule

$\frac{V\frac{\delta u}{\delta t} - U\frac{\delta v}{\delta t}}{V^2}$

$u = 3,\ v = s^2+9,\ \frac{\delta u}{\delta t} = 0,\ \delta v/\delta t = 2s$

$\frac{0 - 3(2s)}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$

vi) $L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_s^\infty \left(\frac{1}{s+1} - \frac{1}{s+2}\right)\delta s$

$= \int_s^\infty \frac{1}{s+1}\delta s - \int_s^\infty \frac{1}{s+2}\delta s$

$= \left[\ln(s+1) - \ln(s+2)\right]_s^\infty$

$= \ln\left[\frac{s+1}{s+2}\right]_s^\infty$

$= 0 - \ln\frac{s+1}{s+2}$

vii) $L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{(s-4)}{(s-4)^2+4}$

viii) $L[t\sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = \frac{-\delta}{\delta x}\left[\frac{2}{s^2+4}\right]$

$u = 2,\ v = s^2+4,\ \frac{\delta u}{\delta s} = 0,\ \delta v/\delta s = 2s$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} = \frac{8-4s}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$$

ix) $L[t^2+4t^3+6] = L(t^2)+L(4t^3)+L(6)$

$$= \frac{2}{s^3} + \frac{8}{s^4} + \frac{6}{s}$$

$L(t^2\cos t)$

$$L(\cos t) = \frac{s}{s^2+1}$$

$$L[t^2\cos t] = -\frac{d}{ds} \times \frac{-d}{ds}\left[\frac{s}{s^2+1}\right]$$

$v = s \quad \frac{dv}{ds} = 1$

$v = s^2+1 \quad \frac{dv}{ds} = 2s$

$$\frac{(s^2+1) - 2s^2}{(s^2+1)(s^2+1)}$$

$$= \frac{d}{ds}\left[\frac{(s^2+1) - 2s^2}{(s^2+1)(s^2+1)}\right]$$

$$= \frac{(-2s^3 - 4s^3 - 6s)}{(s^4+2s^2+1)^2}$$

xii) $L\left[\frac{\sinh 2t}{t}\right]$

$$L[\sinh 2t] = \frac{2}{s^2-4}$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{s}^{\infty} F(s)\,ds$$

$$\int_s^\infty \frac{2}{s^2-4}\,ds$$

$$2\int_s^\infty \frac{1}{s^2-4}\,ds = 2\ln(s^2-4)$$

(4) $L[e^{2t}(t^2+4)]$

$$L(t^2+4) = \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{2t}(t^2+4)] = \frac{2}{(s-2)^3} + \frac{4}{s-2}$$

(3) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$$\Rightarrow \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

$\Rightarrow s-5 = A(s-4)+B(s-3)$

at $s-3=0$, $s=3$

$3-5 = A(3-4)$

$-2 = -A$

$A = 2$

at $s-4=0$, $s=4$

$4-5 = 0+B(4-3)$

$-1 = B$

$B = -1$

$$\Rightarrow L^{-1}\left[\frac{2}{s-3} + \frac{-1}{s-4}\right]$$

$= 2e^{3t} - e^{4t}$

(7) $\frac{2s-6}{(s-2)(s-4)} \Rightarrow \frac{A}{(s-2)} + \frac{B}{(s-4)} = \frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$

$2s-6 = A(s-4)+B(s-2)$

at $s-4=0$, $s=4$

$2(4)-6 = 0+B(4-2)$

$2B = 2$, $B = 1$

at $s-2=0$, $s=2$

$2(2)-6 = A(2-4)+0$

$-2 = -2A$

$A = 1$

$$L^{-1}\left[\frac{1}{s-2}+\frac{1}{s-4}\right]$$

$$= e^{2t} + e^{4t}$$

(iv) $$\frac{5s-8}{s(s-4)} \Rightarrow \frac{A}{s}+\frac{B}{s-4} \Rightarrow \frac{A(s-4)+Bs}{s(s-4)}$$

$5s-8 = A(s-4)+Bs$

at $s=0$

$0-8 = A[-4]$

$A=2$

at $s-4=0, s=4$

$5(4)-8 = B[4]$

$12 = 4B$

$B = 3$