Dalabor Daven John   Assignment 4

15/ENG04/016

Elect - Elect.

Q2

i. $3e^{-4t} - 5e^{4t}$

Recall:

$e^{at} = \frac{1}{s-a}$

$\therefore L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4} = \frac{-2s+32}{(s+4)(s-4)}$

ii. $f(t) = \sin 4t + \cos 4t$

$\sin a = \frac{a}{s^2+a^2}$ $\cos a = \frac{s}{s^2+a^2}$

$= L[\sin 4t] + L[\cos 4t]$

$F(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{(s^2+16)}$

iii. $f(t) = t^3 + 2t^2 - t + 4$

$L\{f(t)\} = L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$= \frac{3!}{s^4} + 2 \cdot \frac{2!}{s^3} - \frac{1}{s} + \frac{4}{s}$

$F(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s} + \frac{4}{s}$

iv. $f(t) = e^{-2t}\cos 5t$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L\{e^{-2t}\cos 5t\} = \frac{(s+2)}{(s+2)^2 + 25}$

v. $f(t) = t \sin 3t$.

$$L\{\sin 3t\} = \frac{3}{s^2+9}$$

$$\therefore L\{t \sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$= (-)\,\frac{-6s}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$$

vi. $f(t) = \dfrac{e^{-t} - e^{-2t}}{t}$

$$\lim_{t \to 0}\left[\frac{e^{-(0)} - e^{-2(0)}}{0}\right] = \frac{1-1}{0} = \frac{0}{0} = \text{undefined}$$

Using L' hopital's rule.

$$\lim_{t \to 0}\left[\frac{-e^{-t} + 2e^{-2t}}{1}\right] = \frac{-1+2}{1} =$$

$$L\{e^{-t} - e^{-2t}\} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$$

vii- $f(t) =$

viii... $f(t) = t\sin$

ix- $f(t) = t^2$

x. $f(t) = e$

Q.

i. $$F(s) = \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$A\Big|_{s=3} = \frac{3-5}{(3-4)} = \frac{-2}{-1} = 2$$

$$B\Big|_{s=4} = \frac{4-5}{4-3} = \frac{1}{1} = 1$$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{1}{s-4}$$

$$f(t) = 2e^{3t} + e^{4t}$$

ii. $$F(s) = \frac{2s-6}{(s-2)(s-4)} = L^{-1}\left[\frac{A}{s-2} + \frac{B}{s-4}\right]$$

$$A\Big|_{s=2} \frac{2(2)-6}{(2-4)} = \frac{4-6}{-2} = \frac{-2}{-2} = 1$$

$$B\Big|_{s=4} \frac{2(4)-6}{4-2} = \frac{8-6}{2} = \frac{2}{2} = 1$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right. = L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right\}$$

$$f(t) = e^{2t} + e^{4t}$$

iii. $$F(s) = \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A\Big|_{s=0} = \frac{5(0)-8}{0-4} = \frac{-8}{-4} = 2$$

vii- $f(t) = e^{4t}\cos 2t$.

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{(s-4)}{(s-4)^2+4}$$

viii- $f(t) = t\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\sin 2t] = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$$=(-)\frac{-4s}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$$

ix- $f(t) = t^2 + 4t^2 + 5$

$$L[f(t)] = L[t^2] + L[4t^2] + L[5]$$

$$= \frac{2!}{s^3} + 4\cdot\frac{2!}{s^3} + \frac{5}{s}$$

$$= \frac{2}{s^3} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{10}{s^3} + \frac{5}{s}$$

x- $f(t) = e^{3t}(t^2+4)$

$$L[t^2+4] = \frac{2!}{s^3} + \frac{4}{s} = \frac{2}{s^3} + \frac{4}{s}$$

$$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

xi. $f(t) = t^2 \cos t$

$$\cos t = \frac{s}{s^2+1}$$

$$L\{t^2 \cos t\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{(s^2+1)-2s^2}{(s^2+1)^2}\right)$$

$$= \frac{(s^2+1)^2 * (2s-4s) - (s^2+1)-2s^2}{(s^2+1)^4}$$

Q2

i. $F(s) = \frac{}{(s-}$

$A\Big|_{s=3} =$

$B\Big|_{s=4} =$

$\frac{s-5}{(s-3)(s-4)}$

$f(t) = 2e$

ii. $F(s) = \frac{2s-6}{(s-2)(s-4)}$

$A\Big|_{s=2} \quad \frac{2(2)}{(}$

$B\Big|_{s=4} \quad \frac{3(4)}{4-}$

$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right.$

$B\Big|_{s=4} = \frac{5(4)-8}{4} = \frac{20-8}{4} = \frac{12}{4} = 3.$

$= L^{-1}\left\{\frac{2}{s} + \frac{3}{s-4}\right\}$

$f(s) = 2 + 3e^{4t}$

iv. $f(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$

$A\Big|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2}$

$= \frac{9-9-4}{(2)^2} = \frac{-4}{4} = -1$

$B\Big|_{s=1} = \frac{1^2-3(1)-4}{(1-3)} = \frac{1-3-4}{-2}$

$= \frac{-6}{-2} = 3$

$C\Big| = \frac{d}{ds}(s-1)^2\Big|_{s=1} = \frac{d}{ds}\frac{s^2-3s-4}{(s-3)}$

$= \frac{d}{ds}$

$= \frac{(s-3)\cdot(2s-3) - (s^2-3s-4)\cdot 1}{(s-3)^2}\Bigg|_{s=1}$

$= \frac{(1-3)(2(1)-3) - (1^2-3(1)-4)}{(1-3)^2}$

$= \frac{-2(-1)-6}{4} = \frac{2-6}{4} = \frac{-4}{4} = -1$

$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} - \frac{1}{s-1}$

$f(t) = -e^{3t} + 3t\,e^{t} - e^{t}$

v. $F(s) = \dfrac{s-5}{s^2+4s+20}$

$s^2+4s+20=0$

$s^2 \pm 4s$

$\dfrac{-4 \pm \sqrt{16-4(1)(20)}}{2}$

$\dfrac{-4 \pm \sqrt{16-80}}{2} = \dfrac{-4 \pm \sqrt{64}}{2}$

$= \dfrac{-4 \pm 8}{2}$

$= -2 \pm 4j$

$(s+2)+4j$

$(s+2)-4j$

$= \dfrac{s-5}{s^2+4s+20} = \dfrac{A}{s^2+4s+20}$

$A(s^2+4s+20) = (s-5)(s^2+4s+20)$

Q1

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

$$(1-x^2)\,y'' - 2xy' + 2y = 0$$

$$\Rightarrow y^{n+2}(1-x^2) - 2ny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$$

rearranging:

$$y^{n+2}(1-x^2) - 2ny^{n+1} - 2xy^{n+1} - n(n-1)y^n - 2ny^n + 2y^n$$

$$y^{n+2}(1-x^2) - 2(ny^{n+1} - xy$$

$$y^{n+2}(1-x^2) - 2y^{n+1}(n-x) - (n(n-1) - 2n + 2)y^n$$

$$y^{n+2}(1-x^2) - 2y^{n+1}(n-x) - (n^2 - n - 2n + 2)y^n$$

$$= y^{n+2}(1-x^2) - 2y^{n+1}(n-x) - (n^2 - n + 2)y^n$$

let $x = 0$

$$y^{n+2} - 2ny^{n+1} - (n^2 - n + 2)y^n = 0$$

$$y^{n+2} = 2ny^{n+1} + (n^2 - n + 2)y^n$$

when $n = 1, 2, 3, 4 \ldots$

$n = 1$

$$y^3 = 2y^2 + (4 - 1 + 2)y$$

$$= 2y^2 + 5y$$