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15/ENG04/057
ELECTRICAL ENGINEERING

1. $[1-x^2]\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$[1-x^2]y'' - 2x \; y' + 2y = 0$

w_1 — $[1-x^2]y''$, w_2 — $2x\,y'$, w_3 — $2y$

for w_1

$u = y''$ $\qquad v = 1-x^2$

$u^n = y^{n+2}$ $\qquad v' = -2x$

$u^{n-1} = y^{n+1}$ $\qquad v'' = -2$

$u^{n-2} = y^n$ $\qquad v''' = 0$

$y^{n+2}[1-x^2] + ny^{n+1} \times -2x + n\frac{(n-1)}{2!}y^n \times -2$

$= y^{n+2} - x^2y^{n+2} + ny^{n+1} \times -2x + n(n-1)y^n \times -1$

$= y^{n+2} - n(n-1)y^n$

For w_2

$-2xy'$

$u = y'$ $\qquad v = -2x$

$u^n = y^{n+1}$ $\qquad v' = -2$

$u^{n-1} = y^n$ $\qquad v'' = 0$

$w_2 = y^{n+1} \times 2x + ny^n \times -2$

$= -2ny^n$

for w_3

$2y.$

$u^n = y^n$ $\quad v = 2$

$v' = 0$

$w_3 = 2y^n$

$w = w_1 + w_2 + w_3$

$= y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n$

$= y^{n+2} + 2y^n - n(n-1)y^n - 2ny^n$

$= y^{n+2} + y^n(2 - n(n-1) - 2n)$

$= y^{n+2} + y^n(2 - n^2 + n - 2n)$

$= y^{n+2} - y^n(2 - n^2 + n - 2n)$

$= y^{n+2} = y^n(-2 + n^2 + n)$

when $n = 0, 1, 2, 3, 4$

$n=0 \; [y^2]_0 = [y]_0 \, [-2+0+0]$

$[y^2]_0 = [y^0][-2]$

$n=1 \; [y^{(3)}]_0 = [y^1]_0 \, [-2+1+1]$

$= [y^1][0]$

$n=2 \; [y^4]_0 = [y^2]_0 \, [-2+2^2+2]$

$[y^4]_0 = [y^2][4] = -8[y^0]_0$

$n = 3 \quad [y^5]_0 = [y^3] \, [-2 + 3^2 + 3]$

$[y^5]_0 = [y^3]_0 \, [10] = 10 \, [0] \, [y^1]_0$

$= 0 \, [y^1]_0$

$n = 4 \quad [y^6]_0 = [y^4]_0 \, [-2 + 4^2 + 4]$

$= [y^4]_0 \, [18] = -8 \times 18 \, [y^0]$

$= -144 \, [y^0]_0$

Maclaurin Series

$y = [y]_0 + x[y']_0 + \frac{x^2}{2!}[y'']_0 + \frac{x^3}{3!}[y''']_0 + \frac{x^4}{4!}[y^{iv}]_0 + \frac{x^5}{5!}[y^{v}]_0 + \frac{x^6}{6!}[y^{vi}]_0$

$= [y]_0 + x[y']_0 + \frac{x^2}{2!}[y'']_0 - \dots$

$= [y]_0 + x[y']_0 + \frac{x^2}{2 \times 1}[y'']_0 - 3 \times \frac{x^4}{4 \times 3 \times 2 \times 1} - 3[y']_0$

$= [y]_0 + x[y']_0 - x^2[y']_0 - \frac{x^4}{3}[y']_0 - \frac{x^6}{5}[y']_0$

$= [y]_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + [y']_0 [x]$

(ii) $L[3e^{-4t}] - L[5e^{4t}]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

(iii) $L[\sin 4t] + L[\cos 4t]$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

(iv) $L[t^3] + L[t^2] - L[t] + L[4]$

$\frac{6}{s^4} + \frac{2}{s^3} - \frac{1}{s^2} - \frac{4}{s}$

(vi) $L[e^{-2t}\cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+5^2}$

$L[e^{-2t}] = \frac{1}{s+2}$

$L[e^{-2t}\cos 5t]$

$\frac{s+2}{(s+2)^2+5^2}$

(v) $L[t \sin 3t]$

$L[\sin 3t] = -\frac{d}{ds}\frac{3}{s^2+3^2}$

$L[t \sin 3t] = \frac{(s^2+3^2)0 - 6s}{(s^2+3^2)^2}$

$-\frac{d}{ds}\frac{3}{s^2+3^2} = -\left[\frac{-6s}{(s^2+3^2)^2}\right]$

$= \frac{6s}{(s^2+9)^2}$

(vi) $L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$

$\lim_{t \to 0}\left[\frac{-e^{-t} + 2e^{-2t}}{1}\right]$

$= e^{-0} + e^{-2(0)}$

$= 1 + 1$

$= 2$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$\int_{\theta = s}^{\infty} \frac{1}{\theta+1} - \frac{1}{\theta+2}$

$= [\ln(\theta+1) - \ln(\theta+2)]_{\theta = s}^{\infty}$

$= \ln(\infty+1) - \ln(s+1) - [\ln(\infty+2) - \ln(s+2)]$

$= \ln(\infty+1) - \ln(s+1) - \ln(\infty+2) - \ln(s+2)$

$= \ln[\infty+1 - (s+1)] - \ln[\infty+2] - s - 2]$

$\ln[\infty - 5] - \ln[\infty - 3]$

$= \ln\left[\frac{\infty - 5}{\infty - 3}\right]$

$= \ln 1$

$= 0$

xii) $e^{4t}\cos 5t$

$L[\cos at] = \frac{a}{s^2+a^2}$

$L[e^{4t}] = \frac{1}{s-4}$

$L[e^{4t}\cos 5t] = \frac{2}{(s^2+4)-4}$

xiii) $L[t \sin 2t]$

$L[\sin 2t] = \frac{2}{s^2+4}$

$-\frac{d}{ds}\frac{2}{s^2+4} = \frac{[s^2+4]0 - 4s}{(s^2+4)^2}$

$= -\left[\frac{-4s}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

xiv) $t^3 + 4t^2 - 15$

$= \frac{6}{s^4} + \frac{8}{s^3} - \frac{5}{s}$

x) $e^{3t}[t^2+4]$

$L[t^2+4] = \frac{2}{s^3} + \frac{4}{s}$

$L[e^{3t}] = \frac{1}{s-3}$

$L[e^{3t}[t^2+4]] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

xi) $t^2 \cos t$

$\cos t = \frac{s}{s^2+1^2}$

$= \frac{s^2+1^2[1] - s(2s)}{(s^2+1^2)^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2}$

$= -\left[\frac{3s^2+1}{(s^2+1)^2}\right]$

$= -\frac{(s^2+1)^2 6s + [3s^2+1](4s)}{(s^2+1)^4}$

$= -\left[\frac{6s^3 - 6s + 12s^3 - 4s}{(s^2+1)^4}\right]$

$= -\frac{(18s^3 - 2s)}{(s^2+1)^2}$

$= \frac{-18s^3 + 2s}{(s^2+1)^2}$

xv) $\frac{\sinh 2t}{t}$

$\left[\frac{2\cosh 2t}{1}\right]$

$t = 0$

$\frac{2\cosh 2(0)}{1} = 2$

$L[2\sinh 2t] = \frac{2}{s^2 - 2^2}$

$\int_{\sigma = s}^{\infty} \frac{2}{\sigma^2 - 2^2}$

$u = \sigma^2 - 2^2$

$\frac{du}{d\sigma} = 2\sigma$

$d\sigma = \frac{du}{2\sigma}$

$\int_{\sigma}^{\infty} \frac{2}{u} \frac{du}{2\sigma}$

$\frac{1}{\sigma} \ln u \Big|_{\sigma}^{\infty}$

$\frac{1}{\sigma} \ln(\sigma^2 - 4)$

$\frac{1}{\infty} \ln[\infty^2 - 4] - \frac{1}{s}(s^2 - 4)$

$= -\frac{1}{s} \ln(s^2 - 4)$

9. $\frac{s-5}{(s-3)(s-4)}$

$\frac{A}{s-3} + \frac{B}{s-4}$

for A $s = 3$ for B $s = 4$

$A[s-4] + B[s-3] = s-5$

for $s = 3$

$A[3-4] + B(3-3) = 3-5$

$-A = -2$

$A = 2$

for $s = 4$

$A[s-4] + B(s-3) = s-5$

$A[4-4] + B[4-3] = 4-5$

$B = -1$

$\therefore \frac{2}{s-3} + \frac{-1}{s-4}$

$2e^{3t} + -e^{4t}$

11. $\frac{2s-6}{(s-2)(s-4)}$

$= \frac{A}{(s-2)} + \frac{B}{s-4}$

for $s = 2$ $s = 4$

$= A[s-4] + B[s-2] = 2s-6$

for $s = 2$

$A[2-4] + B[2-2] = 2(2)-6$

$-2A = -2$

$A = 1$

for $s = 4$

$B[4-2] = 2(4)-6$

$2B = 2$

$B = 1$

$\frac{1}{s-2} + \frac{1}{s-4}$

$= e^{2t} + e^{4t}$

iii] $\frac{5s-8}{s(s-4)}$

$= \frac{A}{s} + \frac{B}{s-4}$

$s=0 \qquad s=4$

$A(s-4) + Bs = 5s-8$

$A(0-4) + B(0) = 5(0) - 8$

$-4A = -8$

$A = \frac{-8}{-4} = 2$

$A(4-4) + B(4) = 5(4) - 8$

$4B = 20 - 8$

$4B = 12$

$B = \frac{12}{4}$

$B = 3$

$\frac{2}{s} + \frac{3}{s-4}$

$2 + 3e^{4t}$

iv] $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s=3 \qquad s=1 \qquad s=1$

for A

$\frac{s^2-3s-4}{(s-1)^2} = \frac{3^2-3(3)-4}{(3-1)^2}$

when $s=3$

$= \frac{9-9-4}{2^2}$

$= \frac{-4}{4}$

$= -1$

for B

$\frac{s^2-3s-4}{(s-3)(s-1)} = \frac{1^2-3(1)-4}{(1-3)(1-1)}$

when $s=1$

$= \frac{1-3-4}{-2 \times 0}$

$= 0$

for C

$\frac{s^2-3s-4}{(s-3)} = \frac{1^2-3(-1)-4}{-1-3}$

when $s=-1$

$= \frac{1+4-4}{-4}$

$= \frac{1}{-4}$

$= -0.25$

$= \frac{-1}{(s-3)} + \frac{0}{(s-1)} - \frac{0.25}{(s-1)^2}$

$= -e^{3t} + 0$

(a) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$= \dfrac{(s-4)(s+1)}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$(s-4)(s+1) = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

when $s=1$

$(1-4)(1+1) = C(1-3)$

$-6 = -2C$

$C = 3$

when $s=3$

$(3-4)(3+1) = A(2)^2$

$-4 = 4A$

$A = -1$

Coefficient of s^2

$1 = A+B$

$B = 1+1$

$= 2$

$\dfrac{-1}{s-3} + \dfrac{2}{s-1} + \dfrac{3}{(s-1)^2}$

$= -e^{3t} + 2e^{t} + 3te^{t}$

(b) $\dfrac{s-5}{s^2+4s+20} = \dfrac{A+Bs}{s^2+4s+20}$

$s-5 = A+Bs$

$s=0$

$-5 = A$

$B = 1$

$F(s) = \dfrac{-5+s}{s^2+4s+20}$

$= \dfrac{s-5}{s^2+4s+20}$

$= \dfrac{s-5}{s^2+4s+4+16}$

$= \dfrac{s-5}{(s+2)^2+16}$

$= \dfrac{s}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2-2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{2}{(s+2)^2+16} - \dfrac{5}{(s+2)^2+16}$

$= \dfrac{s+2}{(s+2)^2+16} - \dfrac{7}{(s+2)^2+16}$

$= e^{-2t}\cos 4t - \dfrac{7}{4} e^{-2t} \sin 4t$