$1^2 - 3(1) - 4 = \cancel{A}(1-1)^2 + B(s-3)(s-1) + C(1-3)$

$\frac{\cancel{-6}}{\cancel{-2}} \quad 3 = \frac{-\cancel{2}}{\cancel{2}}$ $\quad c = 3$

$S = 0$

$0^2 - 3(0) - 4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$

$-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3(3)$

$-4 = -1 + 3B - 9$

$-4 = -10 + 3B$

$-4 + 10 = 3B \qquad B = 2$

$\frac{6}{\cancel{3}}^2 = \frac{\cancel{3}B}{\cancel{3}}$

$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= e^{3t} + 2e^{t} + 3te^{t}$

v) $\frac{s-5}{s^2+4s+20}$

$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$

$s - 5 = As + B$

$A = 1, \quad B = -5$

$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+4)^2+4^2}$

$= e^{5t}\cos 4t$

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+Bs}{s(s-4)}$

$5s-8 = A(s-4)+Bs$

$s=0$ $\qquad$ $s=4$

$= \frac{-8}{\cancel{-4}} \quad \frac{-4}{4}$ $\qquad$ $\frac{12}{4} = B$

$B=3$

$A=2$

$= \frac{2}{s} + \frac{3}{s-4}$

$= 2 + 3e^{4t}$

iv) $\frac{(s^2-3s-4)}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$\frac{s^2-3s-4}{(s-3(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(3-1)+C(s-3)}{(s-3)(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1)+C(s-3)$

$s=3$

$3^2-3(3)-4 = A(3-1)^2$

$\frac{-4}{4} = \frac{4A}{4}$ $\qquad$ $A=-1$

$s=1$

$(s^2+3s+2)^2$

g) $e^{4t}\cos 2t$

$$= \frac{s-4}{(s-4)^2+4}$$

h) $t\sin 2t$

b) $\sin 4t + \cos 4t$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

c) $t^3 + 2t^2 - t + 4$

$$\frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{3}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

d) $e^{-2t} \cos 5t$

$$= \frac{(s+a)}{(s+a)^2+a^2} = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{(s+2)^2+25}$$

e) $t \sin 3t$

$$= \frac{-d}{ds}\left(\frac{a}{s^2+a^2}\right) = \frac{-d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$\frac{d}{ds}\left(3(s^2+9)^{-1}\right) = 3\frac{d}{ds}(s^2+9)^{-1}$$

$$\frac{d}{ds} = \frac{(s^2+9)_0 - 3(3s)}{(s^2+9)^2} = \frac{-9s}{(s^2+9)^2}$$

$y^{n+2} = y^n (n^2 + 3n - 2)$

$(y^{n+2})_0 = (y^n)_0 (n^2 + 3n - 2) \quad n \geq 0$

$n = 0$

$(y^2)_0 = -2 (y)_0$

$n = 1$

$(y^3)_0 = 3 (y')_0$

$n = 2, \quad (y^4)_0 = 8 (y^2)_0$

$n = 3, \quad (y^5)_0 = 48 (y')_0$

$n = 4, \quad (y^6)_0 = 416 (y)_0$

$n = 5, \quad (y^7)_0 = 1824 (y')_0$

$$y = y_0 + x (y')_0 + \frac{x^2}{2!} (y^2)_0 + \frac{x^3}{3!} (y^3 + \frac{x^4}{4!} (y^4) + \frac{x^5}{5!} (y^5)$$

$$y = y_0 + x (y') + \frac{x^2}{2} - 2 (y) + \frac{x^3}{6} 3 (y') + \frac{x^4}{24} - 16 (y) + \frac{x^5}{120} - 1824$$

$$y = (y_0) \left(1 + x^2 - \frac{2}{3} x^4 \right) + (y')_0 \left(x + \frac{x^3}{2} + \frac{76}{5} x^5 + \right)$$

2) 4) $3e^{-4t} - 5e^{4t}$

$$= 3 \cdot \frac{1}{s+4} - 5 \cdot \frac{1}{s-4}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

i) t^3+4t^2+5

$$= \frac{3!}{s^4} + 4\left(\frac{2!}{s^3}\right) + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

j) $(e^{3t} \cdot t^2) + 4$

$$= \frac{2!}{(s+a)^3} + \frac{4}{s}$$

$$= \frac{2}{(s-3)^3} + \frac{4}{s}$$

k) $t^2 \cos t$

$$L(t^2 \cos t) = (-1)^2 \frac{d}{ds}\left(\frac{s}{s^2+1}\right)$$

$$= \frac{(s^2+1) - s(2s)}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

L) $\frac{\sinh 2t}{t} = -1\left[\frac{4s}{(s^2-4)^2}\right] = \frac{4s}{(s^2-4)^2}$

3) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

OGOLO EMMANUEL CLINTON
15/ENG06/049
MECHANICAL ENGINEERING

1) $(1-x^2)\frac{\partial^2 y}{\delta x^2} - 2x\frac{\delta y}{\delta x} + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v^2 + \frac{n(n-1)(n-2)}{6} u^{n-3} v^3 +$

$(1-x^2)y'' - 2xy' + 3y = 0$

For $(1-x^2)y''$, $v = 1-x^2$, $v' = -2x$, $v'' = -2$

$u^n = y^{n+2}$

$= (1-x^2)y^{n+2} - ny^{n+2-1}(-2x) + \frac{n(n-1)}{\cancel{2}} y^{n+2-2}(\cancel{-2}) + 0$

$= (1-x^2)y^{n+2} + 2xny^{n+1} - n(n-1)y^n$

For $-2xy$

$V = -2x$, $v' = -2$, $v'' = 0$

$u^n = y^{n+1}$

$= -2xy^{n-1} + ny^{n+1-1}(-2) + 0$

$= -2xy^{n+1} - 2ny^n$

For $2y$

$v = 2$, $v' = 0$

$y^n = u^n$

$= 2y^n$

$^n = (1-x^2)y^{n+2} + 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n$

at $x = 0$

$= y^{n+2} + 2y^n - (n^2+n)y^n - 2ny^n$

$y^{n+2} = (n^2+n)y^n + 2ny^n - 2y$