Name: Ikuomola Damilola Stephen C

Matric No: 16I ENG061082

Mechanical Engineering.

1. $(1-x^2)\,{}^{dy}/_{dx} - 2x\,{}^{dy}/_{dx} + 2y = 0$

$y^n = u^n v + \cancel{} \; n u^{n-1} v + \frac{n(n-1)}{2!} u^{n-2} v^2 + ---$

$[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)}) - (-2x) + \frac{n(n-1)}{2!} y^n \cdot (-2)] + [y^{(1+n)} - 2x + n y^{n-2} + [2y^n] = 0$

$(1-x^2) y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^n - 2xy^{(1+n)} - 2ny^{(n)} + 2y^n$

when $x=0$

$y^{(2+n)} - n(n-1)y^n - 2ny^{(n)} + 2y^n = 0$

$y^{n+2}\; y^n[-n(n-1) - 2n + 2] = 0$

$y^{n+2} = -(y^n)[-n^2 + n + 2]$

$n=0 \quad y^2 = -y^0 \cdot 2 = -2y^0$

$n=1 \quad y^3 = -y^1(0) = 0$

$n=2 \quad y^4 = -y^2(-4) = 4y^2 = 4(-2y^0) = -8y^0$

$n=3 \quad y^5 = -y^3(-10) = +10y^3 = 0$

$n=4 \quad y^6 = -y^4(-18) = 18y^4 = 18y$

$n=5 \quad y^7 = -y^5(-28) = 28y^5 = 28.0 = 0$

$y = y^0 + xy^1 + \frac{x^2}{2!} y^3 + \frac{x^3}{3!} y^4 + ---$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8y^0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(\cancel{28}\,18)(4)(-2)y^0$

$+ \frac{x^7}{7!}(0)$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3!}y^6 - \frac{x^6}{5}y^0$

$y = y^0\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right) + y^1(x)$

2. $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4} - 5e^{4t}]\quad 3\left\{\frac{1}{s+4}\right\} - 5\left\{\frac{1}{s-4}\right\}$

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1. $(1-x^2)\, dy/dx - 2x\, dy/dx + 2y = 0$

$y^n = u^n v + \cancel{n u^{n-1}}\ n u^{n-1} v + \frac{n(n-1)}{2!} u^{n-2} v^2 + - - -$

$[y^{(2+n)} \cdot (1-x^2) + n y^{(1+n)}) - (-2x) + \frac{n(n-1)}{2!} y^n \cdot (-2)] + [y^{(1+n)} - 2x + n y^{n-2} + [2y^n] = 0$

$(1-x^2) y^{(2+n)} - 2xn y^{(1+n)} - n(n-1) y^n - 2x y^{(1+n)} - 2n y^{(n)} + 2y^n$

when $x = 0$

$y^{(2+n)} - n(n-1) y^n - 2n y^{(n)} + 2y^n = 0$

$y^{n+2}\ y^n [-n(n-1) - 2n + 2] = 0$

$y^{n+2} = -(y^n)[-n^2 + n + 2]$

$n=0 \quad y^2 = -y^0 \cdot 2 = -2y^0$

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$y = y^0 + xy^1 + \frac{x^2}{2!} y^3 + \frac{x^3}{3!} y^4 + - - -$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8y^0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(\cancel{28}\, 18)(4)(-2)y^0$

$+ \frac{x^7}{7!}(0)$

$y = y^0 + xy^1 - x^2 y^0 - \frac{x^4}{3!} y^6 - \frac{x^6}{5} y^0$

$y = y^0 \left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right) + y^1(x)$

2. $3e^{-4t} - 5e^{4t}$

$[3e^{-4} - 5e^{4t}] \quad 3\left\{\frac{1}{s+4}\right\} - 5\left\{\frac{1}{s-4}\right\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\} = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$\frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$ //

iii) $t^3 + 2t^2 - t + 4$

$L\{t^3 + 2t^2 - t + 4\}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1}{s^{1+1}}\right] + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t}\cos 5t$

$L\{\cos 5t\} = \frac{s}{s^2+25}$

$L\{e^{-2t}\cos 5t\} = \frac{(s+2)}{(s+2)^2+25}$ //

v) $L\{t\sin 3t\}$

$-F'(s) = \frac{d}{ds}\left[\sin 3t\right] = \frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$= -(s^2+9)(0) - 3(2s) / (s^2+9)^2$

$= -\left[\frac{-6s}{(s^2+9)^2}\right]$

$= \frac{6s}{(s^2+9)^2}$ //

vi) $L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\}$

$\lim_{t\to 0}\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \frac{1-1}{0} = 0 \to$ indeterminate

$\cancel{L\{f(t)\}}$ By applying L' Hopitals rule

$\lim_{t\to 0}\left\{\frac{-1e^{-t}+2e^{-2t}}{1}\right\} = \left\{\frac{-1+2}{1}\right\} = \frac{1}{1}$

$L\{f(t)\} = e^{-t}-e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$

$= \frac{1}{s+1} - \frac{1}{s+2}$

By replacing s with σ

$\frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\left[\ln(\sigma+1) - \ln(\sigma+2)\right]_0^{\infty}$

$\left[\ln\left(\frac{\sigma+1}{\sigma+2}\right)\right]_0^{\infty} - \ln\left[\frac{\infty+1}{\infty+2} - \frac{s+1}{s+2}\right]$

$-\left[\ln\left(\frac{s+1}{s+2}\right)\right]$

$= \ln\left[\frac{s+2}{s+1}\right]$ //

vii) $e^{4t}\cos 2t$

$L\{\cos 2t\} = \frac{s}{s^2+4}$

$L\{e^{4t}\cos 2t\} = \frac{(s-4)}{(s-4)^2+4}$ //

viii) $t \sin 2t$

$$L\{\sin 2t\} = \frac{2}{s^2+4}$$

$$-F'(s) = \frac{d}{ds}\left[\frac{s}{s^2+4}\right]$$

$$= -\left[\frac{-4s}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2}$$

ix) $t^3 + 4t^2 + 5$

$$L\{t^3+4t^2+5\} = \frac{3!}{s^{2+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x) $e^{3t}(t^2+4)$

~~Let~~ $L\{e^{3t}(t^2+4)\}$

$$L\{(t^2+4)\} = \frac{2!}{s^3} + \frac{4}{s}$$

$$= \frac{2}{s^3} + \frac{4}{s}$$

$$L\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$$

xi $t^2 \cos 2t$

$$L\{\cos t\} = \frac{s}{s^2+1}$$

$$L\{t \cos t\} = \frac{-s^2+1}{(s^2+1^2)^2}$$

$$L\{t^2 \cos t\} = 2s^5 + 4s^3 - 6s \Big/ s^4 + 2s^2 + 1$$

when $s=4$

$B = -1$

when $s=3$

$A = 2$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2e^{3t} - \cancel{\ } e^{4t}$$

ii) $$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$2s-6 = A(s-4) + B(s-2)$

when $s=4$

$8-6 = 2B$

$B = 1$

when $s=2$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$= e^{2t} + e^{4t}$$

iii) $$L\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + B(s)$

when $s=4$

$B = 3$

when $s=0$

$A = 2$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$2 + 3e^{4t}$

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~~iv)~~ v) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$

$s^2-3s-4 = A(s-1)(s-1) + B(s-3) + C(s-3)(s-1)$

when $s=3$

$A=-1$

when $s=1$

$B=3$

when $s=1$

$C=2$

$F(t) = \cancel{\mathcal{L}}\ \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$

$= -e^{-3t} + 3te^{t} + 2e^{t}$

2. $L\left\{\frac{\sinh 2t}{t}\right\} = \ln\left\{\frac{s+2}{s+1}\right\}$