(2)

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$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\,y'' - 2x\,y' + 2y = 0$

$\downarrow$ w_1 $\quad \downarrow$ w_2 $\quad \downarrow$ w_3 using Leibnitz theorem

$w_1 = y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2!}y^n(-2) + 0$

$w_2 = y^{n+1}(-2x) + ny^n(-2)$

$w_3 = 2y^n$

$= y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2!}y^n(-2) + -[y^{n+1}(2x) + ny^n(2)] + 2y^n$

$= y^{n+2}(1-x^2) - ny^{n+1}(-2x) - n(n-1)y^n - y^{n+1}(2x) - ny^n 2 + 2y^n$

at $x=0$

$0 = [y^{n+2}]_0(1) - 0 - [y^n]_0(n-1)n - 0 - n[y^n]_0 2 + 2[y^n]_0$

$0 = [y^{n+2}]_0 - n(n-1)[y^n]_0 - 2n[y^n]_0 + 2[y^n]_0$

$0 = [y^{n+2}]_0 - [y^n]_0(n(n-1) + 2n - 2]$

$[y^{n+2}]_0 = [y^n]_0(n^2 - n + 2n - 2]$

$[y^{n+2}]_0 = [y^n]_0[n^2 + n - 2]$

when $n = 1$

$n: 0\ [y^2]_0 = [y^0]_0(0^2 + 0 - 2] = -2[y^0]_0$

$n: 1\ [y^3]_0 = [y^0]_0[1^2 + 1 - 2] = 0$

$n: 2\ [y^4]_0 = [y^2][4] = -8[y^0]_0$

$n: 3\ [y^5]_0 = [y^3][10] = 0$

$n: 4\ [y^6]_0 = [y^4][18] = -144y^0$

when $n = 5$

$$\{y^7\}_0 = \{y^5\}_0 \, 28 = 0$$

when $n = 6$

$$\{y^8\}_0 = \{y^6\}_0 \, 40 =$$

from maclaurin's series

$$= \{y_0 + x\{y'\}_0 + \frac{x^2}{2!}\{y^2\}_0 + \frac{x^3}{3!}\{y^3\}_0 + \frac{x^4}{4!}\{y^4\}_0$$

$$+ \{y^5\}_0 \frac{x^5}{5!} + \frac{x^6}{6!}\{y^6\}_0 + \cdots$$

$$= y_0 + xy' + \frac{x^2}{2!}\{-2y^0\} + \frac{x^3}{3!}\{0\} + \frac{x^4}{4!}\{-8y^0\}_0$$

$$+ \frac{x^5}{5!}\{0\} + \frac{x^6}{6!}\{-144y^0\}$$

$$= \{y^0\}_0 + x\{y'\}_0 - x^2\{y^0\}_0 - \frac{x^4}{3}\{y_0\} - \frac{x^6}{5}\{y^0\}_0$$

$$+ \cdots$$

$$= \left\{1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} \cdots\right\}\{y^0\}_0 + x\{y'\}_0$$

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15/ENE04/053

2) (i) $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4} = L[3e^{-4t} - 5e^{4t}]$

(ii) $\sin 4t + \cos 4t$

$\frac{4}{s+16} + \frac{s}{s+16} = L[\sin 4t + \cos 4t]$

(iii) $t^3 + 2t^2 - t + 4$

$L[t^3 + 2t^2 - t + 4] = \frac{3!}{s^{3+1}} + 2 \cdot \frac{2!}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t} \cos 5t$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t} \cos 5t) = \frac{s+2}{(s+2)^2+25}$

(v) $t \sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t \sin 3t] = \frac{-d}{ds}\left(\frac{3}{s^2+9}\right)$

$u = 3 \quad \frac{du}{ds} = 0 \qquad v = s^2+9 \quad \frac{dv}{ds} = 2s$

$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{s^2+9(0) - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

$\therefore L[t \sin 3t] = \frac{6s}{(s^2+9)^2}$

vi $\dfrac{e^{-t} - e^{-2t}}{t}$

$$\lim_{t=0} \left[\frac{e^{-t} - e^{-2t}}{t}\right] = \frac{e^{-(0)} - e^{-2(0)}}{0} = \frac{\cancel{1} - \cancel{1}}{0} = 0$$

using L'Hopital's Rule

$$\lim_{t \to 0} \left\{\frac{-e^{-t} + 2e^{-2t}}{1}\right\} = \frac{-e^{(0)} + 2e^{2(0)}}{1} = \frac{-1+2}{1}$$

$$= 1$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{6=s}^{\infty} L\left[e^{-t} - e^{-2t}\right] d6$$

$$= \int_{6=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2} \, d6$$

$$\int_{s}^{\infty} \frac{1}{6+1} d6 - \int_{s}^{\infty} \frac{1}{8+2} d8$$

$$\left[\ln(8+1) - \ln(8+2)\right]_{s}^{\infty}$$

$$\left(\ln \frac{8+1}{8+2}\right)$$

$$\ln\left[\frac{\infty+1}{8+2}\right] - \ln\left[\frac{s+1}{s+2}\right]$$

$$0 - \ln\left[\frac{s+1}{s+2}\right]$$

$$\ln\left[\frac{s+2}{s+1}\right]$$

(vii) $e^{4t} \cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[e^{4t} \cos 2t] = \frac{s-4}{(s-4)^2+4}$$

(ix) $t \sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t \sin 2t] = \frac{-d}{ds}\left(\frac{2}{s^2+4}\right)$$

$u = 2 \quad \frac{du}{ds} = 0 \quad , \quad v = s^2+4 \quad \frac{dv}{ds} = 2s$

$\frac{d}{ds}\left(\frac{2}{s+4}\right) = \frac{0(s^2+4) - 2s(2)}{(s^2+4)^2}$

$= \frac{-4s}{s^2+4}$

$L\{t \sin 2t\} = \frac{-4s}{(s^2+4)^2}$

(x) t^3+4t^2+5

$L\{t^3+4t^2+5\} = \frac{3!}{s^{3+1}} + 4 \cdot \frac{2!}{s^{2+1}} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(xi) $e^{3t}(t^2+4)$

$L\{t^2+4\} = \frac{2!}{s^{2+1}} + \frac{4}{s} = \frac{2}{s^3} + \frac{4}{s}$

$L\{e^{3t}\} = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

(xii) $t^2 \cos t$

$L\{\cos t\} = \frac{s}{s^2+1}$

$L\{t^2 \cos t\} = \frac{-d^2}{ds^2}\left\{\frac{s}{s^2+1}\right\}$

$u = s \quad \frac{du}{ds} = 1 \qquad v = s^2+1 \qquad \frac{dv}{ds} = 2s$

$= \frac{s^2+1(1) - s(2s)}{(s^2+1)^2}$

$= \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$

$\frac{d}{ds}\frac{(1-s^2)}{s^2+1} = u$
$\qquad - v$

$u = 1-s^2 \quad \frac{du}{ds} = -2s \quad , \quad v = (s^2+1)^2$

$\frac{dv}{ds} = 2s \times 2(s^2+1) = 4s(s^2+1)$

$= 4s(s^2+1)$

$\frac{(s^2+1)(-2s) - (1-s^2)\,4s(s^2+1)}{(s^2+1)^4}$

$= \frac{-2s - (1-s^2)4s}{(s^2+1)^3} = -\left[\frac{2s - (1-s^2)4s}{(s^2+1)^3}\right]$

$= \frac{6s - 4s^3}{s^2+1}$

xiii

$\frac{\sinh 2t}{t}$

$\lim_{t \to 0} \left[\frac{\sin 2(0)}{0}\right] = \frac{0}{0}$

using L'Hopital Rule

$\lim_{t \to 0} \left[\frac{2\cos 2(0)}{1}\right] = \frac{2}{1} = 2$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

$L\left[\frac{\sin 2t}{t}\right] = \int_{s=s}^{\infty} \frac{2}{s^2+4}\, ds$

$= \int_s^{\infty} \frac{2}{s^2+4}\, ds = 2\int_s^{\infty} \frac{1}{s^2+4}\, ds$

$= 2\left[\frac{1}{2}\tan^{-1}\frac{s}{2}\right]_s^{\infty}$

$= \left[\tan^{-1}\frac{s}{2}\right]_s^{\infty}$

$= \tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$

$= 90 - \tan^{-1}\frac{s}{2}$

from pythagoras we have:

$= \cancel{\tan^{-1}\frac{s}{2}} + \tan^{-1}\frac{2}{s} - \cancel{\tan^{-1}\frac{s}{2}}$

$= \tan^{-1}\frac{2}{s}$

(3)

i. $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

$s-5 = A(s-4)+B(s-3)$

let $s=3$

$3-5 = A(3-4)$

$-2 = -A$

$A = 2$

let $s=4$

$4-5 = B(4-3)$

$-1 = B$

$= L\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$\frac{A(s-4)+B(s-2)}{(s-2)(s-4)} = \frac{2s-6}{(s-2)(s-4)}$

$2s-6 = A(s-4)+B(s-2)$

let $s=2$

$2(2)-6 = A(2-4)$

$-2 = -2A$

$A = 1$

Let $s=4$

$2(4)-6 = B(4-2)$

$8-6 = 2B$

$B = 1$

$L\left[\frac{1}{s-2}\right] + L\left[\frac{1}{s-4}\right]$

$= e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{5s-8}{s(s-4)} = \frac{A(s-4)+B(s)}{s(s-4)}$

$5s-8 = A(s-4)+B(s)$

let $s=0$

$5(0)-8 = A(0-4)$

$-8 = -4A$

$A = 2$

let $s=4$

$5(4)-8 = B(4)$

$12 = 4B$

$B = 3$

$= L\left[\frac{2}{s} + \frac{3}{s-4}\right]$

$= 2 + 3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} =$

$= \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2+B(s-3)(s-1)+C(s-3)}{(s-3)(s-1)^2}$

$s^2-3s-4 = A(s-1)^2+B(s-3)(s-1)+C(s-3)$

let $s=3$

$9-9-4 = A(3-1)^2$

$-4 = 4A$

$A = -1$

let $s=1$

$1-3-4 = C(1-3)$

$-6 = -2C$

$C = 3$.

for d we expand.

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-2) + 6

2) 7

H (2x)

$y^n\}_0$

$]_0$

the we have

$As^2 - 2As + 1 + Bs^2 - 4s + 3 + Cs - 3$
$= s^2 - 3s - 4$

comparing coefficients

$As^2 + Bs^2 = s^2$

$A + B = 1$

$-1 + B = 1$

$B = 1 + 1 = 2$

$\mathcal{L}^{-1}\left\{\frac{-1}{(s-3)} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right]$

$= e^{3t} + 2e^t + 3e^{t}t$

$= e^{3t} + 2e^t + 3te^t$

$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$

$s - 5 = As + B$

let $s = 0$

$-5 = B$

let $s = 1$

$1 - 5 = A(1) + B$

$-4 = A + B$

$-4 = A + (-5)$

$A = 5 - 4$

$A = 1$

$= \frac{s-5}{s^2-4s+20}$

$\frac{s-5}{s^2-4s+20} = \frac{s-5}{s^2-4s+20}$

$\frac{s^2-5}{s^2-4s+4+16} = \frac{s^2-5}{(s-2)^2+16}$

1

$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$w_1 \quad w_2 \quad w_2$

$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16}$

$= \frac{s-5}{(s+2)^2+16} =$

$\frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$\frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$\frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$

$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2}$

$\frac{s+2}{(s+2)^2+4^2} - \left(\frac{4}{(s+2)^2+4^2}\right)\cdot\frac{7}{4}$

$= e^{-2t}\cdot\cos 4t - \frac{7}{4}\sin 4t\, e^{-2t}$

$= e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$

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