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16/ENG04/071
ELECT/ELECT

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$\omega_1 \quad \omega_2 \quad \omega_3$

ω_1

$U = y''$ $\quad V = 1-x^2$

$U^n = y^{(n+2)}$ $\quad V' = -2x$

$U^{n-1} = y^{(n+1)}$ $\quad V'' = -2$

$U^{n-2} = y^{(n)}$ $\quad V''' = 0$

$y_n = y^{(n+2)}\cdot(1-x^2) + ny^{(n+1)}(-2x) + y^{(n)}\cdot\frac{(-2)n(n-1)}{2!}$

ω_2

$U = y'$ $\quad V = -2x$

$U^n = y^{(n+1)}$ $\quad V' = -2$

$U^{n-1} = y^n$ $\quad V'' = 0$

$y_{n\omega_2} = y^{(n+1)}\cdot(-2x) + ny^n(-2)$.

ω_3

$U = y$ $\quad V = 2$

$U^n = y^n$ $\quad V' = 0$

$y_{n\omega_3} = y^n\cdot 2$

$y_n = y_{n\omega_1} + y_{n\omega_2} + y_{n\omega_3} = 0$

at $x = 0$

$y_n = y^{(n+2)} \cdot n(n-1)y^n - 2ny^n + 2y^n$.

$y^{(n+2)} - n(n-1)y^n - 2ny^n + 2y^n = 0$.

$y^{(n+2)} + y^n[-n^2+n-2n+2] = 0$.

$y^{(n+2)} + y^n[-n^2-n+2] = 0$.

$y^{(n+2)} = -y^n[-n^2-n+2]$.

$y^{(n+2)} = y^n[n^2+n-2]$ — recurrence equation

$n=0 \; (y^{(2)})_0 = y^0(-2) = -2(y^0)_0$.

$n=1 \; (y^{(3)})_0 = y^1(0) = 0$.

$n=2 \; (y^{(4)})_0 = y^2(4) = 4(y^2)_0$
$= 4(-2)(y^0)_0 = -8(y^0)_0$.

$n=3 \; (y^{(5)})_0 = 0$

$n=4 \; (y^{(6)})_0 = y^4(18) = 18(y^4)_0$
$= 18\cdot -8(y^0)_0 = -144($

$n=5 \; (y^{(7)})_0 = y^5(28) = 28(y^5)_0 = 0$.

$n=6 \; (y^{(8)})_0 = y^6(40) = 40(y^6)_0 =$
$40\cdot -144(y^0)_0 = -5760($

$n=7 \; (y^{(9)})_0 = y^7(54) = 54(y^7)_0 = 0$

$(y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(-2)(y^0)_0 + \frac{x^3}{3!}($

$+\frac{x^4}{4!}(-8(y^0)_0) + \cdots + \frac{x^5}{5!}(0)$

$+\frac{x^6}{6!}(-144)(y^0)_0 + \frac{x^7}{7!}(0) +$

$\frac{x^8}{8!}(-5760)(y^0)_0 + \frac{x^9}{9!}(0)$.

$= (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^0)_0$
$- \frac{1}{3}x^4(y^0)_0 - \frac{1}{5}x^6(y^0)_0 - \frac{1}{7}($

Question 2.

i) $3e^{-4t} - 5e^{4t} = f(t)$.

$F(s) = \frac{3}{s+4} - \frac{5}{s-4}$ //

ii) $\sin 4t + \cos 4t = f(t)$

$\mathcal{L}[f(t)] = \mathcal{L}[\sin 4t] + \mathcal{L}[\cos$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

iii) $t^3 + 2t^2 - t + 4 = f(t)$.

$F(s) = \frac{3!}{s^4} + \frac{2\cdot 2!}{s^3} + -\frac{1}{s^2}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + 4/s$ //.

(iv) $e^{-2t}\cos 5t = f(t)$.

$F(s) = \frac{s+2}{(s+2)^2+25}$ //

(v) $t \sin 3t$

$F(s) = L[t\sin 3t]$

$= \frac{-d}{ds} \cdot \frac{3}{s^2+9}$

$= \frac{-d}{ds} \cdot 3(s^2+9)^{-1}$

$= -[-3 \cdot 2s(s^2+9)^{-2}]$.

$= -[-6s(s^2+9)]$.

$= 6s[s^2+9]^{-2}$

$= \cancel{6s}\ \frac{6s}{(s^2+9)^2}$ //.

(vi) $\frac{e^{-t} - e^{-2t}}{t}$.

$\lim_{t\to 0} \left[\frac{e^{-t} - e^{-2t}}{t}\right]$

$\lim \left[\frac{-e^{-t} + 2e^{-2t}}{1}\right] = -1+2 = 1$

$\cancel{L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \int_{\sigma=s}^{s} \frac{1}{\sigma+2}}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} F(\sigma)\, d\sigma$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}$

$= \ln(\sigma+1)_s^{\infty} - \ln(\sigma+2)_s^{\infty}$

$= \ln[\infty+1-s-1] - \ln[\infty+2-s-2]$

$= \ln[\infty - s] - \ln[\infty - s]$

$= \ln\left[\frac{\infty - s}{\infty - s}\right] = \ln(1)$

$= 0$ //.

(vii) $e^{4t}\cos 2t$

$L[e^{4t}\cos 2t]$

$= \frac{s-4}{(s-4)^2+4}$

(viii) $t\sin 2t$.

$F(s) = L(t\sin 2t)$

$= \frac{-d}{ds} \cdot \frac{2}{s^2+4}$

$= \frac{-d}{ds} \cdot 2(s^2+4)^{-1}$

$= -[2(-1) \cdot 2s(s^2+4)^{-2}]$.

$= -[-2(s^2+4)^{-2}]$.

$= \frac{2}{(s^2+4)^2}$ //.

(ix) $t^3 + 4t^2 + 5$

$L[t^3 + 4t^2 + 5]$

$F(s) = \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + 5/s$

$= \frac{6}{s^4} + 8/s^3 + 5/s$.

(x) $e^{3t}(t^2+4)$

$L[t^2+4] = 2/s^3 + 4/s$.

$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + 4/$

(xi) $t^2 \cos t$

$L[\cos t]$

$= \frac{s}{s^2+1}$

$L[t^2 \cos t]$

$= (-1)^2 \frac{d^2}{ds^2} \cdot \frac{s}{s^2+1}$

$= \frac{d^2}{ds^2} \cdot \frac{s}{s^2+1}$

$= s(s^2+1)^{-1}$

$= s(-2s(s^2+1)^{-2}) + (s^2+1)^{-1}$

$\frac{d}{ds} = \frac{-2s^2}{(s^2+1)^2} + \frac{1}{s^2+1}$

$F(s) = -2s^2[-4s(s^2+1)^{-3}] + (s^2+1)^{-2} \cdot -4s$

$+ \cdots + -2s(s^2+1)^{-2}$

$= \frac{8s^3}{(s^2+1)^3} - \frac{4s}{(s^2+1)^2} - \frac{2s}{(s^2+1)^2}$

$= \frac{8s^3(s^2+1) - 4s - 2s}{(s^2+1)^2}$

$= \frac{8s^3(s^2+1) - 6s}{(s^2+1)^2}$

(xii) $\frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

$L\left[\frac{\sinh 2t}{t}\right] = \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2-4} \, d\sigma$

Let $u = \sigma^2 - 4$

$\frac{du}{d\sigma} = 2\sigma \Rightarrow d\sigma = \frac{du}{2\sigma}$

$= \int_{\sigma=s}^{\infty} \frac{2}{u} \cdot \frac{du}{2\sigma}$

$= \frac{1}{\sigma} \int_{\sigma=s}^{\infty} \frac{1}{u} \, du$

$= \frac{1}{\sigma} \ln u = \frac{1}{\sigma} \ln(\sigma^2-4) \Big|_{\sigma}^{\infty}$

$= \frac{1}{\infty} \ln(\infty - 4) - \frac{1}{s} \ln(s-4)$

$= 0 - \frac{1}{s} \ln(s-4) = -\frac{1}{s} \ln(s-4) = \ln($

Question 3.

i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

When $s=4$.

$-1 = B = \Rightarrow B = -1$

When $s=3$.

$-2 = -A$

$A = 2$

$= \frac{2}{s-3} - \frac{1}{s-4}$

$f(t) = 2e^{3t} - e^{4t}$ //.

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$.

$s=2$.

$-2 = -2A$

$A = 1$

$s = 4$

$2 = 2B \Rightarrow B = 1$

$= \frac{1}{s-2} + \frac{1}{s-4}$

$f(t) = e^{2t} + e^{4t}$ //

iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

$s = 0$

$-8 = -4A$

$A = 2$

$s = 4$

$12 = 4B$

$B = 3$

$= \frac{2}{s} + \frac{3}{s-4}$

$f(t) = 2 + 3e^{4t}$.

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$.

$= \frac{(s-4)(s+1)}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$.

$(s-4)(s+1) = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$s = 1$

$(1-4)(1+1) = C(1-3)$

$-6 = -2C$

$C = 3$

$s = 3$

$(3-4)(3+1) = A(2)^2$

$-4 = 4A$

$A = -1$

Coefficients of s^2

$1 = A + B$

$B = 1+1$

$= 2$

$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$.

$= -e^{3t} + 2e^{t} + 3te^{t}$.

v) $\frac{s-5}{s^2+4s+20} = \frac{A+Bs}{s^2+4s+20}$

$s-5 = A+Bs$

$s = 0$

$-5 = A$

$B = 1$

$F(s) = \frac{-5+s}{s^2+4s+20}$

$= \frac{s-5}{s^2+4s+4+16}$

$= \frac{s-5}{(s+2)^2+16}$

$= \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$.

$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$.

$= \frac{s+2}{(s+2)^2+16} - \frac{7}{(s+2)^2+16}$

$F(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$

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i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$ $y=2$

$y' + 3y = e^{-2t}$.

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)[s+3] - 2 = \frac{1}{s+2}$.

$(s+3)y(s) = \frac{1}{s+2} + 2$.

$y(s)[s+3] = \frac{1+2(s+2)}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$.

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$.

$2s+5 = A(s+3) + B(s+2)$.

$s = -3$.

$-1 = -B$.

$B = 1$

$s = -2$

$1 = A$.

$= \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$.

ii) $3\frac{dy}{dt} - 6y = \sin 2t$ at $t=0$ $y=1$

$3y' - 6y = \sin 2t$.

$3[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$.

$y(s)[3s-6] - 3 = \frac{2}{s^2+4}$

$y(s)[3s-6] = \frac{2}{s^2+4} + 3$.

$y(s) = \frac{2+3(s^2+4)}{(s^2+4)(3s-6)}$.

$= \frac{2+3s^2+12}{(s^2+4)(3s-6)}$.

$= \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$.

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$s=2$

$26 = C(4+4)$

$C = \frac{26}{8} = 13/4$.

Coefficient of s^2

$3 = 3B + C$

$\frac{3-C}{3} = B$

$B = \frac{3 - 13/4}{3} = -1/12$

Coefficients of s.

$0 = 3A - 6B$

$A = \frac{6B}{3} = 2B = 2 \times -1/12$

$= -1/6$.

$y(s) = \frac{-1/6 - 1/12 s}{s^2+4} + 13/4 \cdot \frac{1}{3s-6}$

$y(s) = -1/6 \cdot \frac{1}{s^2+4} - \frac{1}{12} s \cdot \frac{1}{s^2+4}$

$+ 13/\ldots \frac{1}{\ldots}$

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$$y(s) = -\frac{1}{12} \cdot \frac{2}{s^2+4} - \frac{1}{12} \cdot \frac{s}{s^2+4} + \frac{13}{12} \frac{1}{s-2}$$

$$y(t) = -\frac{1}{12} \sin 2t - \frac{1}{12} \cos 2t + \frac{13}{12} e^{2t}$$

$$y(t) = \cancel{1/2} -\frac{1}{12} \left[\sin 2t + \cos 2t - 13e^{2t} \right]$$