15|ENG03|030

SMOOTH SIMON

CIVIL ENGINEERING

ASSIGNMENT

1) $(1-x)^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$

$(1-x)^2 y^2 - 2xy' + 2y = 0.$

$y^n = U^n V + nU^{n-1} V^1 + \frac{n(n-1)}{2!} U^{n-2} V^2 + \dots$

$\left[y^{(2+n)} (1-x^2) + ny^{(n+n)} \frac{(-2x) + n(n-1)y^n}{2!} \cdot (-2)\right]$

$+ \left[y^{(1+n)} - 2x + ny^n - 2 + [2y^n]\right] = 0.$

$(1-x^2)y^{n+2} - 2x \cdot ny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

Let $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n[-n(n-1) - 2n + 2] = 0.$

$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0.$

$y^{n+2} + y^n[-n^2 - n + 2] = 0.$

$y^{n+2} = -(y^n)_0 [-n^2 - n + 2].$

$n = 0 : y^2 = -y^0 \cdot 2 = -2y.$

$n = 1 : y^3 = -y^1 \cdot [0] = 0$

$n = 2 : y^4 = -y^2 \cdot [-4] = 4y^2 = 4(-2y^0) = -8^0.$

$n = 3 : y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4 : y^6 = -y^4 \cdot [-18] = 10y^4 = 18 \cdot 4 - 2y^0$

$n = 5 : y^7 = -y^5 \cdot [-28] = 28(y^3) = 28 \cdot 0 = 0.$

$y = y^0 + xy^1 + \frac{x^2 y^4}{2!} + \frac{x^3}{3!} y^3 + \dots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0.$

$+ \frac{x^5}{5!}(0) + \frac{x^3}{6!}(18)y \quad (-2)y^\circ + \frac{x^7}{7!}(0).$

$y = y^\circ + xy' - x^2y^\circ - \frac{x^4}{3\times 1} y^\circ - \frac{x^6}{5} y^\circ$

$y = y^\circ \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y'_0 [x].$

(i) $3e^{-4t} - 5e^{4t}$

$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

(v)

(ii) $\sin 4t + \cos 4t$

$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t].$

$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+10} = \frac{4+s}{(s^2+6)}.$

(iii) $t^3 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right]$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(vi)

(iv) $e^{-2t} \cos 5t$

$L(\cos 3t) = \frac{s}{s^2+a^2}$

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$$= \frac{s}{s^2+5^2} = \frac{5}{s^2+25}$$

$$L[e^{-2t} \cos 5t] = \frac{s+2}{[s+2]^2+25}$$

(3) $t \sin 3t$

$$L(\sin 3t) = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2}$$

$$= \frac{3}{s^2+9}$$

$$L(t \sin 3t) = -f'(s)$$

$$\frac{V\frac{du}{ds} - U\frac{dv}{ds}}{V^2}$$

$U = 3$ $\qquad$ $V = s^2+9$

$\frac{du}{ds} = 0$ $\qquad$ $\frac{dv}{ds} = 2s$

$$= \frac{[s^2+9] \cdot 0 - 3[2s]}{[s^2+9]^2}$$

$$= \frac{-6s}{[s^2+9]^2}$$

$$-f'(s) = -1\left[\frac{-6s}{(s^2+9)^2}\right]$$

$$= \frac{6s}{(s^2+9^2)}$$

(4) $\frac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[f(t)] = e^{-t} - e^{-2t} = \frac{1}{s+a} - \frac{1}{s+a}$$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$F(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\int F(s) = L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2} \, d\sigma$$

$$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} \, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} \, d\sigma$$

$$\ln[\sigma+1] - \ln[(\sigma+2)]_s^{\infty}$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} =$$

$$= \ln\left[\frac{\alpha+1}{(\sigma+2)} = \frac{s+1}{(s+2)}\right] = -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{(s+2)}{(s+1)}\right]$$

(vii) $e^{4t} \cos 2t$

$$L(\cos 2t) = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t} \cos 2t] = \frac{s-4}{[s-4]^2+4}$$

(viii) $t \sin 2t$

$$L[t \sin 2t] = \frac{-d}{ds}[F(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$F(s) = \frac{2}{s^2+4}$$

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2} \qquad u = 2, \quad \frac{du}{ds} = 0$$

$$= \frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

(ix) $t^3 + 4t^2 + 5$

$$L[t^3+4t^2+5]$$

$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

(x) $e^{3t}(t^3+4)$

let $x = t^2+4$

$$L[e^{3t}x]$$

$$L[x] = L[t^3+4]$$

$$= L[t^2] + L[4]$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

$$= \frac{2!}{s^3} + \frac{4}{s}$$

$$L[e^{3t}] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

(xi) $t^2 \cos t$

$$L[t^2 \cos t] = t^2 L(\cos t)$$

$$f(s) = L(\cos t) = \frac{s}{s^2+1^2}$$

$$f(s) = \frac{s}{s^2+1^2}$$

$F'(s)$

$u = s \qquad v = s^2+1$

$\frac{du}{ds} = 1 \qquad \frac{dv}{ds} = 2s$

$$= \frac{[s^2+1^2]1 - 2s[s]}{[s^2+1^2]^2}$$

$$= \frac{s^2 + 1^2 - 2s^2}{[s^2 + 1^2]^2}$$

$$= \frac{-s^2 + 1}{[s^2 + 1^2]^2}$$

$$-f''(s) = \frac{-d}{ds}\left[\frac{s^2 - 1}{(s^2 + 1^2)^2}\right]$$

$$U = s^2 - 1 \qquad V = [s^2 + 1]^2$$

$$\frac{du}{ds} = 2s \qquad \frac{dv}{ds} = 1/s\,[s^2 + 1]^2$$

$$= \frac{[s^2 + 1]^2 \cdot 2s - [s^2 - 1][4s^2 + 4s]}{[s^2 + 1]^2}$$

$$= \frac{[2s^2 - 4s^2 + 2s] - [4s^2 - 4s]}{L[(s^2 + 1)^2]}$$

$$= \frac{2s^2 - 4s^3 + 2s - 4s^3 + 4s}{(s^2 + 1)^2}$$

$$= \frac{-2s^3 - 4s^3 + 6s}{(s^2 + 1)^2}$$

$$= \frac{-2s^3 - 4s^2 + 6s^1}{s^4 + 2s^2 + 1}$$

$$f''(s) = \frac{-d}{ds}\left[\frac{-2s^3 \cdot 4s^3 + 6s}{s^4 + 2s^2 + 1}\right]$$

$$f''(s) = \frac{2s^3 + 4s^2 - 6s}{s^4 + 2s^2 + 1}$$

(xii) $\frac{s - 5}{(s-3)(s-4)}$

$$L^{-1}\left\{\frac{s - 5}{(s-3)(s-4)}\right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s - 5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s = 4$

$$4 - 5 = A(4-4) + B(4-3)$$

$$-1 = B(1)$$

$$B = -1$$

Assuming $s = 3$

$3 - 5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$\therefore \dfrac{s-5}{(s-3)(s-4)} = \dfrac{2}{s-3} + \dfrac{-1}{s-4}$

$= \dfrac{2}{s-3} - \dfrac{1}{s-4}$

$= 2\left[\dfrac{1}{s-3}\right] - \left[\dfrac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

(viii) $\dfrac{2s-6}{(s-2)(s-4)}$

$L^{-1}\left[\dfrac{2s-6}{(s-2)(s-4)}\right]$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{(s-2)} + \dfrac{B}{(s-4)}$

$2s - 6 = A(s-4) + B(s-2)$

Assuming $s = 4$

$(2(4) - 6) = A(4-4) + B(4-2)$

$8 - 6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

Assuming $s = 2$

$(2(2) - (6)) = A(2-4) + B(2-2)$

$4 - 6 = -2A + 0$

$-2 = -2A$

$A = 1$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$L^{-1}\left[\dfrac{2s-6}{(s-2)(s-4)}\right]$

$= e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)}$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

Assuming $s = 4$

$$5(4)-8 = A(4-4) + B(4)$$

$$20-8 = A(0) + 4B$$

$$12 = 4B$$

$$B = 3$$

Assuming $s = 0$

$$5(0)-8 = A(0-4) + B(0)$$

$$-8 = -4B$$

$$B = 3$$

Assuming $s = 0$

$$s(0)-8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$L^{-1}\left[\frac{s^2-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

(v) $\frac{s-5}{s^2+4s+20}$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right]$$

$$F(s) = \frac{s-5}{s^2+4s+20} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)+16} - \frac{2}{(s+2)^2+16}$$

$$f(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$

(iv) $\dfrac{s^2 - 3s - 4}{(s-3)(s-1)^2}$

$$f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \left.\frac{s^2 - 3s - 4}{(s-1)^2}\right|_{s=3} = \frac{3^2 - 3(3) - 4}{(3-1)^2} = -1$$

$$B = \left.\frac{s^2 - 3s - 4}{s-3}\right|_{s=1} = \frac{1^2 - 3(1) - 4}{1-3} = 3$$

$$C = \frac{d}{ds}\left[\frac{s^2 - 3s - 4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3s - 4)}{(s-3)^2}$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{3t} + 3t e^{t} + 2e^{t}$$

$$= e^{t}[3t + 2] - e^{3t}.$$