Bello Miriam Ayuba
15/scu46010
Chemical Engineering

1. $[1-x^2]\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$[1-x^2]y'' - 2xy' + 2y = 0$

$W_1 \quad W_2 \quad W_3$

For W_1

$y^n = U^nV + nU^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2$

$U = y'' \quad U^n = y^{n+2} \quad U^{n-1} = y^{n+1} \quad U^{n-2} = y^n$

$V = (1-x^2), \quad V' = -2x \quad V^2 = -2 \quad V^3 = 0$

$y^n = y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2}y^n(-2)$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n.$

For W_2

$U = y' \quad U^n = y^{n+1} \quad U^{n-1} = y$

$V = x \quad V' = 1 \quad V^2 = 0$

$y^n = x.y^{n+1} + ny^n.1 -$

$= xy^{n+1} + ny^n.$

For W_3

$y^n = y^n$

Combining

$y^n = W_1 + W_2 + W_3$

$y^n = [1-x^2]y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2(xy^{n+1} + ny^n) + 2y^n$

if $x = 0$

$y^{n+2} + y^n(2-2n-n^2+n) = 0$

$y^{n+2} + y^n(2-n-n^2) = 0$

$y^{n+2} = (n^2+n-2)y^n.$

$n \geq 0$

for $n = 0, \quad y^3 = 0 - 2y^{(0)}$

$n = 1, \quad y^3 = 0y^{(1)} = 0$

For $n = 2$, $y^4 = 4y^2 = 4(-2)(y^0)$

$n = 3$ $\quad y^5 = 10y^3 = 10(0)(y^1) = 0$

$n = 4$ $\quad y^6 = 18y^4 = 18(4)(-2)(y^0)$

$n = 5$ $\quad y^7 = 28y^5 = 28(10)(0)(y^1) = 0$

$$y_n = y^{(0)} + x(y^{(1)}) + \frac{x^2}{2!}(y^2) + \frac{x^3}{3!}(y^3) + \frac{x^4}{4!}(y^4)$$

$$+ \frac{x^5}{5!}(y^5) + \frac{x^6}{6!}(y^6) + \frac{x^7}{7!}(y^7)$$

$$y_n = y^0 + xy^1 + \frac{x^2}{2}(-2)(y^0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^0)$$

$$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^0) + \frac{x^7}{7!}(0)$$

$$y_n = y^0 + xy^1 - x^2(y^0) - \frac{x^4}{3}(y^0) - \frac{x^6}{5}(y^0)$$

$$y_n = y^0\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right) + y^1(x)$$

2. $(3e^{-4t} - 5e^{4t}) = \cancel{\frac{3}{s-(-4)} - \frac{5}{s+4} = \frac{3}{s+4} - \frac{5}{s+4} = -2}$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii $\sin 4t + \cos 4t = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

iii $t^3 + 2t^2 - t + 4 = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1}{t^{1+1}} + \frac{4}{s}$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{t^2} + \frac{4}{s}$$

iv $e^{-2t}\cos 5t = \frac{s+2}{(s-(-2))^2+5^2} = \frac{s+2}{(s+2)^2+5^2}$

$$= \frac{s+2}{s^2+4s+4+25} = \frac{s+2}{s^2+4s+29}.$$

v $t\sin 3t = (-1)^n \cdot \frac{d^n}{dx^n}[f(t)] = -1 \cdot \frac{d}{dx}(\sin 3t)$

$= -1 \cdot \frac{d}{dx}\left(\frac{3}{s^2+3^2}\right)$ $du = 0$, $dv = 2s$

$= -\left(\frac{-6s}{(s^2+9)^2}\right) = \frac{6s}{s^4+18s^2+81}$

vi $\frac{e^{-t}-e^{-2t}}{t} = t^{-1}(e^{-t}-e^{-2t})$

$= (-1)^1 \frac{dx}{d}\left[\frac{1}{s+1} - \frac{1}{s+2}\right]$

$= -\frac{dx}{d}\left[\frac{s+2-s-1}{(s+1)(s+2)}\right] = -\frac{dx}{d}\left[\frac{1}{s^2+3s+2}\right]$

$du = 0$

$dv = 2s+3$

$= -\left[\frac{2s+3}{(s^2+3s+2)^2}\right]$

vii $e^{4t}\cos 2t = \frac{s-4}{[(s-4)^2+2^2]} = \frac{s-4}{s^2-8s+16+4} = \frac{s-4}{s^2-8s+20}$

viii $t\sin 2t = [-1]^1 \frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$ $du = 0$, $dv = 2s$

$= -1\left[\frac{-4s}{(s^2+2^2)^2}\right] = \frac{4s}{(s^4+8s^2+16)}$

ix $t^3+4t^2+5 = \frac{3!}{s^{3+1}} + \frac{4[2!]}{s^{2+1}} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x $e^{3t}[t^2+4] = e^{3t}t^2 + 4e^{3t}$

$= \frac{2!}{(s-3)^{1+1}} + \frac{4}{s-3} = \frac{2}{(s-3)^2} + \frac{4}{s-3}$

xi $t^2\cos t = (-1)^2 \cdot \frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right]$ $du = 1$, $dv = 2s$

$= \frac{d}{dx}\left(\frac{1-s^2}{s^2+1}\right)$ $du = -2s$, $dv = 2s$

$= \left[\frac{-2s^3-2s-2s+2s^2}{(s^2+1)^2}\right] = \frac{-4s}{(s^2+1)^2}$

$$\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-2^2) - \ln s$$
$$= \frac{1}{2}\ln(s^2-4) - \ln s$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$
$$s-5 = A(s-4) + B(s-3)$$
$$3-5 = A(3-4) + B(3-3)$$
$$A = \frac{-2}{-1} = 2.$$
$$4-5 = A(4-4) + B(4-3)$$
$$B = -1$$
$$= \frac{2}{s-3} - \frac{1}{s-4}$$
$$L^{-1}\left\{\frac{2}{s-3} - \frac{1}{s-4}\right\} = 2e^{3t} - e^{4t}.$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$
$$2s-6 = A(s-4) + B(s-2)$$
$$2(2)-6 = A(2-4) + B(2-2)$$
$$-2 = -2A$$
$$A = 1$$
$$2(4)-6 = A(4-4) + B(4-2)$$
$$2 = 2B$$
$$B = 1$$
$$= \frac{1}{s-2} + \frac{1}{s-4}$$
$$= e^{2t} + e^{4t}.$$

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$
$$A(s-4) + Bs = 5s-8$$
$$A(0-4) + B*0 = 5(0)-8$$
$$A = 2$$
$$A(4-4) + 4B = 5(4)-8$$
$$B = \frac{12}{4} = 3$$
$$= \frac{2}{s} + \frac{3}{s-4}$$
$$= 2 + 3e^{4t}.$$

iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$s^2-3s-4 = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$A+B = 1$

$-C-2A-4B = -3$

$A+3B-3C = -4$

$1^2-3(1)-4 = C(1-3)$

$-6 = -2C$

$C = 3$

$A+3B-3(3) = -4$

$A+3B = -4+9 = 5$

$A+3B = 5$

$-A+B = 1$

$2B = 4$

$B = 2$

$A+B = 1$

$A\ (1-B = 1-2 = -1$

$= L^{-1}\left\{-\dfrac{1}{s-3} + \dfrac{2}{s-1} + \dfrac{3}{(s-1)^2}\right\}$

$= -e^{3t} + 2e^{t} + \dfrac{3}{2}t^2e^{t}$

v) $\dfrac{s-5}{s^2+4s+20} = \dfrac{s-5}{s^2+4s+4+16}$

$= \dfrac{s-5}{(s+2)^2+16} = \dfrac{s-5}{(s+2)^2+4^2}$

$= e^{2t}\cos 4t$