Assignment 4

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$u = y''$, $u' = y'''$, $u'' = y^{iv}$, $u''' = y^{v}$

$v = 1-x^2$, $v' = -2x$, $v'' = -2$, $v''' = 0$

$w^n = (1-x^2)y^{n+2} + n\, y^{n+1} \cdot -2x +$

$\frac{n(n-1)}{2!} y^{n} \cdot -2 + \frac{n(n-1)(n-2)}{3!} y^{n-1} \cdot 0$

$= (1-x^2)y^{n+2} - 2xy^{n+1}n -$

$n(n-1)y^n$

$P = -2xy'$

$u = y'$, $u' = y''$, $u'' = y'''$, $u''' = y^{iv}$

$v = -2x$, $v' = -2$, $v'' = 0$

$P^n = -2x\, y^{n+1} + n\, y^{n} \cdot -2 + \frac{n(n-1)}{2}\, y^{n-1} \cdot 0$

$= -2xy^{n+1} - 2ny^n$

$Q = 2y$

$Q^n = 2y^n$

$\Rightarrow (1-x^2)y^{n+2} - 2xy^{n+1}n -$

$n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$\Rightarrow (1-x^2)y^{n+2} - 2xny^{n+1} - (n^2-n)y^n$

$- 2xy^{n+1} - 2ny^n + 2y^n$

when $x = 0$

$\Rightarrow y^{n+2} - (n^2-n)y^n - 2ny^n$

$+ 2y^n$

2. (i) $L\{3e^{-4t} - 5t^{2}\} = \frac{3}{s+4} - [illegible]$

(ii) $L\{3\sin 4t + 5\cos 4t\} = \frac{4}{s^2+16} + \frac{5}{s^2+16}$

$= \frac{4+5}{s^2+16}$

(iii) $L\{t^3 + 2t^2 - t + 4\} = \frac{3!}{s^{3+1}}$

$+ \frac{2\{2!\}}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $L\{e^{-2t}\cos 5t\} = \frac{s+2}{s^2+4s+29}$

(v) $L\{t\sin 3t\} = (-1)^n \cdot \frac{d^n}{dx^n}\{f(s)\}$

$= -1 \cdot \frac{d}{ds}\left[\frac{3}{s^2+3^2}\right]$

$du = 0$

$dv = 2s$

$= \left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

(vi) $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \left\{\frac{\frac{1}{s+1} - \frac{1}{s+2}}{s^{[illegible]}}\right\}$

$= \left[\frac{1}{s+1} - \frac{1}{s+2}\right]$ [illegible]

$L\{e^{4t}\cos 2t\} = \frac{s}{(s-4)^2+2^2}$

$= \frac{s}{s^2-8s+20}$

(b) $L\{t\sin 2t\} = -\frac{d}{dx}\left[\frac{2}{s^2+2^2}\right]$

$du = 0$

$dv = 2s$

$= -1\left[\frac{-4s}{(s^2+2^2)^2}\right] = \frac{4s}{(s^2+4)^2}$

$t^3+4t^2+5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

$e^{3t}[t^2+4] = t^2e^{3t} + 4e^{3t}$

$= \frac{2!}{(s-3)^{2+1}} + \frac{4}{s-3}$

$= \frac{2}{(s-3)^3} + \frac{4}{s-3}$

$L\{t^2\cos t\} = (-1)^2\frac{d^2}{dx^2}\left[\frac{s}{s^2+1}\right]$

$= \frac{d}{dx}\left(\frac{d}{dx}\left[\frac{s}{s^2+1}\right]\right)$

$du = 1 \quad dv = 2s$

$= \frac{d}{dx}\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$

$= \frac{d}{dx}\left[\frac{1-s^2}{(s^2+1)^2}\right]$ $du = -2s$, $dv = 4s^3+4s$

$= \left[\frac{-2s^5-4s^3-2s-4s+4s^5}{[(s^2+1)^2]^2}\right]$

$= \left[\frac{2s^5-4s^3-6s}{(s^2+1)^4}\right]$

$L\left\{\frac{1-\cos 2t}{t}\right\} = \frac{1}{2}\ln(s^2+4) - \ln s$

3 (i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

$3-5 = A(3-4) + \cancel{B(3-3)}$

$\Rightarrow A = 2$

$4-5 = B(4-3) \Rightarrow B = -1$

$L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right] = 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2(2)-6 = A(2-4) \Rightarrow A = 1$

$2(4)-6 = B(4-2) \Rightarrow B = 1$

$L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right] = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

$5(0)-8 = A(0-4) \Rightarrow 2$

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$S(4) - 8 = B(4) \Rightarrow 2$

$L^{-1}\left[\frac{2}{S} + \frac{3}{S-4}\right] = 2 + 3e^{4t}$

iv) $\frac{S^2 - 3S - 4}{(S-3)(S-1)^2} = \frac{A}{S-3} + \frac{B}{S-1} + \frac{C}{(S-1)^2}$

$S^2 - 3S - 4 = A(S-1)^2 + B(S-3)(S-1) + C(S-3)$

$3^2 - 3(3) - 4 = A(3-1)^2 \Rightarrow -1$

$1^2 - 3(1) - 4 = C(1-3) \Rightarrow 3$

$S^2 - 3S - 4 = (S^2 - 2S + 1)A + (S^2 - 4S + 3)B + (S-3)C$

$-2A - 4B + C = -3$

$-2(-1) - 4(B) + 3 = -3$

$-4B = -3 - 3 - 2 \Rightarrow B = 2$

$L^{-1}\left[\frac{-1}{S-3} + \frac{2}{S-1} + \frac{3}{(S-1)^2}\right]$

$= e^{-3t} + 2e^{t} + 3te^{t}$

v) $\frac{S-5}{S^2+4S+20} = \frac{S-5}{S^2+4S+4+16}$

$\Rightarrow \frac{S-5}{(S+2)^2 + 4^2} = L(e^{-2t} - 7)$

$= (e^{-2t} - 7)\cos 4t$

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