BABATUNDE [illegible]

15/ENG06/052

MECHANICAL ENGINEERING

ENG 381

ASSIGNMENT IV B

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)\frac{d^2y}{dx^2} = (1-x^2)y''$

$u = y^2 \quad u^n = y^{(n+2)}$

$v = 1-x^2 \quad v' = -2x \quad v^2 = -2 \quad v^3 = 0$

$w^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}v^2}{2!} + \frac{n(n-1)(n-2)u^{(n-3)}v^3}{3!}$

$= y^{(n+2)}\cdot(1-x^2) + ny^{(n+1)}\cdot -2x + \frac{n(n-1)y^n\cdot 2}{2} + 0$

$w^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$W_2 = -2x\frac{dy}{dx} = -2xy'$

$u = y' \quad u^n = y^{n+1}$

$v = -2x \quad v' = -2 \quad v^2 = 0$

$= y^{(n+1)}\cdot -2x + ny^n\cdot -2 + 0$

$= -2xy^{(n+1)} - 2ny^n$

$W_2 = 2y$

$u = y \quad u^n = y^n$

$v = 2 \quad v' = 0$

$y^n \cdot 2 + 0$

$= 2y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$

$= (1-x^2)y^{(n+2)} - 2xy^{(n+1)}(n+1) - y^n(n^2-n+2n-2)$

$= (1-x^2)y^{n+2} - (n+1)2xy^{(n+1)} - (n^2+n-2)y^n$

at $x = 0$

$= (1-0^2)y^{(n+2)} - (n+1)2(0)y^{(n+1)} - (n^2+n-2)y^n$

$y^{(n+2)} = (n^2+n-2)y^n$

$y^{n+2}_0 = (n^2+n-1)y^n$

at $n = 0$.

$[y^2]_0 = -2[y^0]_0$

at $n = 1$

$[y^3]_0 = 0$

at $n = 2$

$[y^4]_0 = 4[y^2]_0 = 4 \times -2[y^0]_0 = -8[y^0]_0$

at $n = 3$

$[y^5]_0 = 10[y^3]_0 = 10 \times 0 = 0$

at $n = 4$

$[y^6]_0 = 18[y^4]_0 = 18 \times -8[y^0]_0 = -144[y^0]_0$

$y = y_0 + x(y')_0 + \frac{x^2}{2!}[y^2]_0 + \frac{x^3}{3!}[y^3]_0 + \ldots \frac{x^r}{r!}(y^r)_0$

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$= y_0 + x(y')_0 + \frac{x^2}{2!} \cdot 2[y^0]_0 + \frac{x^3}{3!}\,0 + \frac{x^4}{4!} \cdot 8[y^0]_0 + \frac{x^5}{5!}\,0 + \frac{x^6}{6!} \cdot 194[y$

$= y^0_0 + x(y')_0 - x^2(y^0)_0 - \frac{x^4}{3}(y^0)_0 - \frac{x^6}{5}(y^0)_0$

$y_0\left(1 - x^2 - x^4/3 - x^6/5\right) + x(y')_0$

2\. i. $L[3e^{-4t} - 5e^{4t}]$

$L[3e^{-4t}] - L[5e^{4t}]$

$\frac{3}{s+4} - \frac{5}{s-4}.$

ii. $L\{s$

ii. $L[\sin 4t + \cos 4t]$

$L[\sin 4t] + L[\cos 4t]$

$\frac{4}{s^2+16} + \frac{4}{s^2+16} = \frac{4+s}{s^2+16}.$

iii. $L[t^3 + 2t^2 - t + 4]$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} + 2\left(\frac{2!}{s^3}\right) - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv. $L[e^{-3t}\cos 5t]$

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$= \left[\frac{1}{2}\,\frac{\tan^{-1}(s)}{2}\right]^{\infty} = \left[\frac{\tan^{-1}(s)}{2}\right]^{\infty}_{s}$

$\tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$

$= \tan^{-1} s/2$

$\left(\tan^{-1} s/2\right)^{-1} = \tan^{-1} 2/s$

3. Given the [illegible]

$: \frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-4}\right]$

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

at $s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = A(0) + B(1)$

$-1 = 0 + B \quad ; \quad B = -1.$

at $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = -A$

$A = 2$

$L^{-1}\left[\frac{2}{s-3} + \frac{1}{s-4}\right]$

$x(t) = 2e^{3t} - e^{4t}$

$\frac{2s-6}{(s-2)(s-4)}$

$L^{-1}\left[\frac{A}{(s-2)} + \frac{B}{(s-4)}\right]$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

at $s=4$

$2(4)-6 = A(4-4) + B(4-2)$

$2 = 2B \quad B = 1.$

at $s=2$

$2(2)-6 = A(2-4) + B(2-2)$

$-2 = -2A \; ; \; A = 1$

$x(t) = L^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$

$x(t) = e^{2t} + e^{4t}$

iv)

$\frac{5s-8}{s(s-4)}$

$L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

at $s=4$

$12 = 4B$

$B = 3$

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$= \ln\left[\frac{s+1}{s+2}\right]$

$\ln\left[\frac{s+1}{s+2}\right]^{-1} = \ln\left[\frac{s+2}{s+1}\right]$

vii) $L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -1\frac{d}{ds}\left[\frac{2}{s^2+4}\right]\frac{u}{v}$

$u = 2 \quad du = 0$

$v = s^2+4 \quad dv = 2s$

$-1\left[\frac{0-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$

ix. $t^3 + 4t^2 + 5$

$L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \left[\frac{5}{s}\right]$

$= 6/s^4 + 8/s^3 + 5/s$

x. $t^2\cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$L[t^2\cos t] = (-1)^2\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$

$\frac{d}{ds}\left[\frac{s}{s^2+1}\right]\frac{u}{v}$

$u = s \quad du = 1 \quad v = s^2+1 \quad dv = 2s$

$-1\left[\frac{1-2s^2}{(s^2+1)^2}\right] = \frac{1-2s^2}{(s^2+1)^2}$

xi. $e^{3t}(t^2+4)$

$L[t^2e^{3t} + 4e^{3t}]$

$L[t^2e^{3t}] = 2!$

$= \frac{2}{(s-3)^3}$

$L[4e^{3t}] = \frac{4}{s-3}$

$\therefore e^{3t}(t^2+4) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

xii. $\frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

$L\left[\frac{\sinh 2t}{t}\right] = \int_{s}^{\infty}\frac{2}{s^2-4} = 2\int_{s}^{\infty}\frac{1}{\sigma^2-4}$

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at $s = 0$

$s(0) - 8 = A(s-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$x(t) = 2 + 3e^{4t}$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2}$$

$$L^{-1}\left[\frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}\right]$$

$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

at $s = 3$

$-4 = 4A$

$A = -1$

at $s = 1$

$-6 = -2C$

$C = 3$

$s^2 - 3s - 4 = As^2 - 2As + A + Cs - C3 + Bs^2 - B4s + B3$

$A + 3B - 3C = -4$

$-1 + 3B - 3(3) = -4$

$3B = 6, \quad B = 2,$

Subst

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$x(t) = -e^{3t} + 2e^{t} + 3te^{t}$

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$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{(s+2)^2+25}$

. $t \sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t \sin 3t] = -1 \frac{d}{ds}\left[\frac{3}{s^2+9}\right]$ u/v

$u = 3 \quad du = 0$

$v = s^2+9 \quad dv = 2s$

$-1\left[\frac{0-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

. $e^{-t} - e^{-2t}$

$L(e^{-t} - e^{-2t})$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_{s}^{\infty} 1/\sigma+1 - 1/\sigma+2$

$\ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\frac{\sigma+1}{\sigma+2}\right]$

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