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1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

For W_1

$U = y''$ $\quad V = 1-x^2$

$U^n = y^{(n+2)}$ $\quad V' = -2x$

$U^{(n-1)} = y^{(n+1)}$ $\quad V'' = -2$

$U^{(n-2)} = y^n$ $\quad V''' = 0$

For W_2

$U = y'$ $\quad V = 2x$

$U^n = y^{(n+1)}$ $\quad V' = 2$

$U^{(n-1)} = y^n$ $\quad V'' = 0$

For W_3

$U = y$ $\quad V = 2$

$U^n = y^n$ $\quad V' = 0$

$\therefore W_1 + W_2 + W_3 = 0$

$\therefore y^{(n+2)}(1-x^2) + n\,y^{(n+1)} \cdot -2x + \frac{n(n-1)}{2!} y^n \cdot -\cancel{2} + y^{(n+1)} \cdot 2x + n\,y^n \cdot 2$
$+ y^n \cdot 2 = 0$

$\therefore (1-x^2)y^{(n+2)} - 2xn\,y^{(n+1)} - n(n-1)y^n + 2x\,y^{(n+1)} + 2n\,y^n + 2y^n = 0$

Let $x = 0$

$\therefore (1-0)y^{(n+2)} - \cancel{2(0)n\,y^{(n+1)}} - n(n-1)y^n + \cancel{2(0)y^{(n+1)}} + 2n\,y^n + 2y^n = 0$

$y^{(n+2)} - n(n-1)\,y^n + 2n\,y^n + 2y^n = 0$

$y^{(n+2)} + (n^2+n)\,y^n + 2n\,y^n + 2y^n = 0$

$y^{(n+2)} + (n^2 + n + 2n + 2)\,y^n$

$y^{(n+2)} = -(-n^2 + 3n + 2)\,y^n$

$y^{(n+2)} = (n^2 - 3n - 2)\,y^n$

When

$n = 0;\ y^2 = (0 - 2(y^0)$.

$n=1;\ y^3 = -4(y^1)_0$

$n=2;\ y^4 = -4(y^2)_0 = (-4)(-2)(y^0)_0$

$n=3;\ y^5 = -2(y^3)_0 = (-2)(-4)(y^1)_0$

$n=4;\ y^6 = 2(y^4)_0 = (2)(-4)(-2)(y^0)_0$

$n=5;\ y^7 = 8(y^5)_0 = (8)(-2)(-4)(y^1)_0$

$n=6;\ y^8 = 16(y^6)_0 = (16)(2)(-4)(-2)(y^0)_0$

Substituting

$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(-2)(y^0)_0 + \frac{x^3}{3!}(-4)(y^1)_0 + \frac{x^4}{4!}(-4)(-2)(y^0)_0$

$+ \frac{x^5}{5!}(-2)(-4)(y^1)_0 + \frac{x^6}{6}(2)(-4)(-2)(y^0)_0 + \frac{x^7}{7}(8)(-2)(-4)(y^1)_0$

$+ \frac{x^8}{8}(16)(2)(-4)(-2)(y^0)_0$

$= (y^0)_0 + x(y^1)_0 - x^2(y^0)_0 - \frac{2x^3}{3}(y^1)_0 + \frac{x^4}{3}(y^0)_0 + \frac{x^5}{5\times 3}(y^1)_0$

$+ \frac{x^6}{5\times 3\times 3}(y^0)_0 + \frac{4x^7}{7\times 3\times 5} + \frac{x^8}{}$

2i) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} = L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t - \cos 4t$

$L\{\sin 4t\} - L\{\cos 4t\}$

$= \frac{4}{s^2+4^2} - \frac{s}{s^2+4}$

iii) $t^3 + 2t^2 - t + 4$

$L\{t^3 + 2t^2 - t + 4\}$

$= \frac{3!}{S^{3+1}} + 2\frac{2!}{S^{2+1}} - \frac{1}{S^{1+1}} + \frac{4}{S}$

$= \frac{6}{S^4} + \frac{4}{S^3} - \frac{1}{S^2} + 4/s$

iv) $e^{-2t} \cos 5t$

$L\{\cos 5t\}$

$= \frac{S}{S^2+5^2} = \frac{S}{S^2+25}$

$\therefore L\{e^{-2t} \cos 5t\}$

$= \frac{S+2}{(S+2)^2+25}$

v) $t \sin 3t$

$\therefore L\{\sin 3t\}$

$= \frac{3}{S^2+3^2}$

$= \frac{3}{S^2+9}$

$\therefore L\{t \sin 3t\} = \frac{-d}{ds}\left\{\frac{3}{s^2+9}\right\}$

Using quotient rule

$\therefore U = 3 \quad V = S^2+9$

$du = 0 \quad dv = 2S$

$\therefore \frac{V\,du/ds - U\,dv/ds}{V^2}$

$= \frac{(S^2+9)(0) - 3(2s)}{(S^2+9)^2}$

$= \frac{-6s}{(S^2+9)^2} = \frac{6s}{(S^2+9)^2}$

vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

$\therefore L\{e^{-t} - e^{-2t}\} = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty}\left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)d\sigma$$

$$= \int_{\sigma=s}^{\infty}\frac{1}{\sigma-3}\,d\sigma - \int_{\sigma=s}^{\infty}\frac{1}{\sigma+2}\,d\sigma$$

$$= \left[\ln(\sigma-3) - \ln(\sigma+2)\right]_s^{\infty}$$

$$= \ln\left[\frac{\sigma-3}{\sigma+2}\right]_s^{\sigma}$$

$$= \left[\ln\frac{\sigma-3}{\sigma+2} - \ln\left(\frac{s-3}{s+2}\right)\right]$$

$$= \ln 1 - \ln\left(\frac{s-3}{s+2}\right)$$

$$= 0 - \ln\left(\frac{s-3}{s+2}\right)$$

$$= -\ln\left(\frac{s-3}{s+2}\right)$$

vii) $e^{4t}\cos 2t$

$$L\{\cos 2t\} = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$\therefore L\{e^{4t}\cos 2t\}$

$$= \frac{s+4}{(s\mp 4)^2+4}$$

viii) $t \sin 2t$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$L\{t \sin 2t\} = -\frac{d}{ds}\left\{\frac{2}{s^2+4}\right\}$$

Using quotient rule

$U = 2 \quad V = (s^2+4)$

$du = 0 \quad dv = 2s$

$$\therefore \frac{V\frac{du}{ds} - U\frac{dv}{ds}}{V^2}$$

$$= \frac{\cancel{(s^2+4)(0)} - 2(2s)}{(s^2+4)^2}$$

$$= (-)\frac{-4s}{(s^2+4)^2} = \frac{4s}{(s^2-4)^2}$$

ix) $t^3 + 4t^2 + 5$

$$L\{t^3 + 4t^2 + 5\}$$

$$= \frac{3!}{s^{3+1}} + 4\frac{2!}{s^{2+1}} + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x) $e^{3t}(t^2+4)$

$$L\{t^2+4\}$$

$$= \frac{2!}{s^{2+1}} + \frac{4}{s}$$

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$= \frac{2}{s^3} + \frac{4}{s}$

$\therefore L\{e^{3t}(t^2+4)\}$

$= \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

x) $t^2 \cos t$

$L\{\cos t\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$

$\therefore L\{t^2 \cos t\} = -\frac{d^2}{ds^2}\left\{\frac{s}{s^2+1}\right\}$

$\therefore t' = \frac{d}{ds}\left\{\frac{s}{s^2+1}\right\}$

$= u = s \qquad v = s^2+1$

$du = 1 \qquad dv = 2s$

Using quotient rule

$\frac{V\,du/ds - U\,dv/ds}{V^2}$

$= \frac{(s^2+1)1 + s(2s)}{(s^2+1)^2} = \frac{(s^2+1) + 2s^2}{(s^2+1)^2}$

$\therefore t'' \left\{\frac{(s^2+1) + 2s^2}{(s^2+1)^2}\right.$

$\therefore U = (s^2+1) + 2s^2 \qquad V = (s^2+1)^2$

$du = 2s + 4s \qquad dV = 2 \times 2(s^2+1)$

$= 4(s^2+1)$

$$\frac{V\frac{du}{ds} - U\frac{dv}{ds}}{V^2}$$

$$= \frac{(s^2+1)^2(2s+4s) - [(s^2+1)+2s^2)(4(s^2+1))}{((s^2+1)^2)^2}$$

xii) $\frac{\sinh 2t}{t}$

$\lim_{t \to 0} \left\{\frac{\sinh 2t}{t}\right\} = \frac{\sinh 2(0)}{0} = \frac{0}{0}$

L'Hopital's rule

$\lim_{t \to 0} \left\{\frac{2\cosh 2t}{1}\right\} = \frac{2\cosh 2(0)}{1} = \frac{2}{1} = 2$

$L\{\sinh 2t\} = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$

$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{t=s}^{\infty} \frac{2}{s^2-4}\, ds$

$= \cancel{2} \int \frac{2}{s^2-4}\, ds$

$= \ln\left[\frac{2}{-s^2 4}\right]_s^{\infty}$

$= \ln\left[\frac{2}{s^2-4}\right] - \ln\left[\frac{2}{s^2-2}\right]$

$= \ln\left[\frac{2}{s^2-4}\right]$

$= \ln\left[\frac{2}{s^2-4}\right]^{-1}$

$= \ln\left[s^2 - \frac{4}{2}\right.$

3a) $\cancel{\frac{s+5}{(s-3)(s-4)}}$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$s-5 = A(s-4) + B(s-3)$$

$$s-5 = As - 4A + Bs - 3B$$

$\therefore A + B = 1$

$-4A - 3B = -5$

$\therefore \cancel{B} = A = 1 - B$

$-4(1-B) - 3B = -5$

$-4 + 4B - 3B = -5$

$B = -5 + 4$

$B = -1$

$\therefore A = 2$

$$\therefore \frac{2}{(s-3)} + \frac{-1}{(s-4)}$$

$= 2e^{3t} \cancel{+(-} - e^{4t})$

3b)

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

$$2s-6 = As - 4A + Bs - 2B$$

$A + B = 2$

$-4A - 2B = -6$

$A = 2 - B$

$\therefore -4(2-B) - 2B = -6$

$-8 + 4B - 2B = -6$

$2B = -6 + 8$

$2B = 2$

$B = \cancel{2/2}\ 1$

$B = 1$

$A = 2 - 1$

$A = 1$

$$\therefore \frac{1}{(s-2)} + \frac{1}{(s-4)}$$

$e^{2t} + e^{4t}$

c)

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$$

$5s - 8 = A(s-4) + Bs$

$5s - 8 = As - 4A + Bs$

$A + B = 5$

$-4A = -8$

$A = \frac{\cancel{-8}^{2}}{\cancel{-4}_{1}}$

$A = 2$

$\therefore B = 5 - 2$

$B = 3$ //

$$\therefore \frac{2}{s} + \frac{3}{(s-4)}$$

$= 2 + 3e^{4t}$ //

d) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$

$s^2-3s-4 = As^2-2As+A+Bs^2-4Bs+3B+Cs-3C$

$1 = A+B$

$-3 = -2A-4B-C$

$-1 = A+3B-3C$

$A = 1-B$

$-3 = -2(1-B)-4B+C$

$-3 = -2+2B-4B+C$

$-1 = -2B+C \quad ---- ①$

$-1 = 1-B+3B-3C$

$-2 = +2B-3C$

$-1 = -2B+C$

$\underline{-2 = 2B-3C}$

$-3 = -2C$

$C = 3/2$

$-1 = -2B+3/2$

$2B = 3/2 + 1$

$2B = 5/2$

$B = 5/4$

$A = 1 - 5/4$

$A = -1/4$

$$= \frac{-1}{4(s-3)} + \frac{5}{4(s-1)} + \frac{3}{2(s-1)^2}$$

$$= \frac{-1}{4} e^{3t} + \frac{5}{4} e^{t} + \frac{3}{2} e^{t}$$