

$$2) (1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y^{(0)} = 0$$

$$\text{let } u^0 = y'', u^1 = y''', u^2 = y^{(4)}$$

$$v^0 = (1-x^2), v^1 = -2x, v^2 = -2$$

$$w_1 = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \frac{(-2)(n)(n-1)}{2!} y^n$$

$$w_2 = -(y^{(n+1)})(2x) + ny^{(n)}(2)$$

$$y^{(n+2)}(1-x^2) - 2xn y^{(n+1)} - n(n-1)y^n - 2(y^{(n+1)} - 2ny^{(n)} + 2y^{(n)}) = 0$$

$$0 = y^{(n+2)}(1-x^2) + y^{(n+1)}(2xn - 2x) + y^{(n)}(-2n + 2 - n(n-1))$$

when $x=0$

$$y^{(n+2)}(1-0) + y^{(n+1)}(0-0) + y^{(n)}(-n^2 - n + 2) = 0$$

$$y^{(n+2)} + y^{(n)}(-n^2 - n + 2) = 0$$

$$y^{(n+2)} = y^{(n)}(n^2 + n - 2) = 0 \quad \text{recurrent solution}$$

$$\text{when } n=0 \quad y_{(0)}^{(2)} = y_{(0)}^0(0+0-2) = y_{(0)}^0(-2) = -2y_{(0)}^0$$

$$n=1 \quad y_{(0)}^{(3)} = y_{(0)}^1(1^2+1-2) = y_{(0)}^1(0) = 0$$

$$n=2 \quad y_{(0)}^{(4)} = y_{(0)}^2(2^2+2-2) = y_{(0)}^2(4) = 4 \times -2y_{(0)}^0$$

$$n=3 \quad y_{(0)}^{(5)} = y_{(0)}^3(3^2+3-2) = y_{(0)}^3(10) = 10 \times 0$$

$$n=4 \quad y_{(0)}^{(6)} = y_{(0)}^4(4^2+4-2) = y_{(0)}^4(18) = 18 \times 4 \times -2y_{(0)}^0$$

$$n=5 \quad y_{(0)}^{(7)} = y_{(0)}^5(5^2+5-2) = y_{(0)}^5(28) = 28 \times 10 \times 0$$

$$n=6 \quad y_{(0)}^{(8)} = y_{(0)}^6(6^2+6-2) = y_{(0)}^6(40) = 40 \times 18 \times 4 \times -2y_{(0)}^0$$

$$y = y_{(0)}^0 + x y_{(0)}^1 + \frac{x^2}{2!} \cdot y_{(0)}^2 + \frac{x^3}{3!} \cdot y_{(0)}^3 + \frac{x^4}{4!} \cdot y_{(0)}^4 + \frac{x^5}{5!} \cdot y_{(0)}^5 + \frac{x^6}{6!} \cdot y_{(0)}^6$$

$$+ \frac{x^7}{7!} \cdot y_{(0)}^7 + \frac{x^8}{8!} \cdot y_{(0)}^8$$

$$2.) \quad 3e^{-4t} - 5e^{4t} = f(t)$$

$$\mathcal{L}\{f(t)\} = F(s)$$

$$F(s) = \mathcal{L}\{3e^{-4t}\} - \mathcal{L}\{5e^{4t}\}$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

$$y = y'''' + xy'' + x^2(-2y') + \frac{x^4}{1} (4x - 2y') + \frac{x^6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} (18 \times 4x - 2y') + \frac{x^8}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} (40 \times 18 \times 4x - 2y')$$

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$$2.) \quad f(t) = 3e^{-4t} - 5e^{4t}$$

$$F(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

$$ii.) \quad \sin 4t + \cos 4t = f(t)$$

$$F(s) = \frac{4}{s^2+16} + \frac{5}{s^2+16}$$

$$iii.) \quad f(t) = t^3 + 2t^2 - t + 4$$

$$F(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$iv.) \quad e^{-2t} \cos 5t = \frac{s}{(s+2)^2 + 25}$$

$$v.) \quad t \sin 3t = -\frac{d}{ds} \left(\frac{3}{s^2+9} \right) = \frac{(s^2+9)(0) - (3)(2s)}{(s^2+9)^2}$$

$$= \frac{6s}{(s^2+9)^2}$$

$$vi.) f(t) = e^{-t} - e^{-2t}$$

$$F(s) = \int_0^{\infty} \frac{1}{b+t} db - \int_0^{\infty} \frac{1}{b+2} db$$

$$= (\ln(b+1) - \ln(b+2)) \Big|_0^{\infty}$$

$$= (\ln(\frac{b+1}{b+2})) \Big|_0^{\infty}$$

$$= (-\ln(\frac{s+1}{s+2}) + (\ln(\frac{\infty}{\infty})))$$

$$= \ln(s+2) - \ln(s+1)$$

$$G.S. = P.D. \cdot F.C.F.$$

$$y = 1 \cdot 2$$

$$\text{let } b^2 + 4 = u \quad \frac{du}{db} = 2b ; db = \frac{du}{2b}$$

$$\int_0^{\infty} \frac{1}{b} \frac{du}{2b} = \frac{1}{2} \int_0^{\infty} \frac{du}{u} = \frac{1}{2} \ln u \Big|_0^{\infty}$$

$$vii.) e^{4t} \cos 2t = \frac{s}{(s^2+4)^2 + 4}$$

$$viii.) f(t) = t \sin 2t$$

$$F(s) = -\frac{d}{ds} \left(\frac{2}{s^2+4} \right) = \frac{(s^2+4)(0) - (2)(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

$$ix.) f(t) = t^3 + 4t^2 + 5$$

$$F(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$x.) f(t) = e^{3t}(t^2+4) = t^2 e^{3t} + 4e^{3t}$$

$$F(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

$$xi.) f(t) = t^2 \cos t$$

$$F(s) = (-1)^2 \frac{d^2}{ds^2} \left(\frac{s}{s^2+1} \right) = \frac{(s^2+1)(1) - (s)(2s)}{(s^2+1)^2}$$

$$= \frac{d}{ds} \left(\frac{-s^2+1}{(s^2+1)^2} \right) = \frac{(s^2+1)^2(-2s) - (-s^2+1)(2s)(2s)}{(s^2+1)^4}$$

$$xii.) \sinh 2t$$

$$F(s) = \int_0^{\infty} \frac{2}{b^2+4} db \quad \text{let } u = b^2+4 \quad \frac{du}{db} = 2b \quad db = \frac{du}{2b}$$

$$= \frac{1}{2b} \int_0^{\infty} \frac{2}{u} du = \left(\frac{\ln u}{b} \right) \Big|_0^{\infty}$$

$$= \left(\frac{\ln(b^2+4)}{b} \right) \Big|_0^{\infty}$$

$$3i.) \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s=3 \Rightarrow 3-5 = A(3-4) + 0$$

$$-2 = -A; A=2$$

$$\text{if } s=4; 4-5 = B(4-3)$$

$$-1 = B$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4} = 2e^{3t} - e^{4t}$$

$$3ii) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$(s-2)(s-4) \quad s-2 \quad s-4$$

$$\text{if } s=2; 2(2)-6 = A(2-4)$$

$$-2 = -2A; A=1$$

$$\text{if } s=4; 2(4)-6 = B(4-2)$$

$$2 = 2B; B=1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$$

$$(s-2)(s-4) \quad s-2 \quad s-4$$

$$3iii) \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$s(s-4) \quad s \quad s-4$$

$$\text{if } s=0; 5(0)-8 = A(0-4)$$

$$-8 = -4A; A=2$$

$$\text{if } s=4; 5(4)-8 = B(4)$$

$$12 = 4B; B=3$$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$$

$$s(s-4) \quad s \quad s-4$$

$$3iv) f(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$(s-3)(s-1)^2 \quad s-3 \quad (s-1)^2 \quad (s-1)^2$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3) + C(s-3)(s-1)$$

$$s^2-3s-4 = As^2-2As+A + Bs-3B + Cs^2-4Cs+C$$

$$s^2-3s-4 = s^2(A+C) + s(-2A+B-4C) + A-3B+C$$

$$A+C=1$$

$$-2A+B-4C=-3$$

$$A-3B+C=-4$$

$$A = \frac{1}{3}, B = \frac{5}{3}, C = \frac{2}{3}$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{3(s-3)} + \frac{5}{3(s-1)^2} + \frac{4}{3(s-1)}$$

$$= \frac{1}{3} (-e^{3t} + 5te^t + 4e^t)$$

$$v) \frac{s-5}{s^2+4s+20} = \frac{Ax+B}{s^2+4s+20}$$

$$s-5 = Ax+B$$

$$As+B=0$$

$$4A+B=1$$

$$20A+20B=-5$$

$$B = -A$$

$$4A + 4(-A) = 1$$