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13/ENG04/007
Chemical Engineering
Process Dynamics and Control Assignment

(1) Applying Routh's stability Approach

$$y(s) = \frac{G_p(s)\, G_f(s)\, G_c(s)}{1 + G_p(s)\, G_f(s)\, G_c(s)\, G_m(s)} \cdot y_{sp}(s)$$

For a P only controller, $G_c = k_c$ (Proportional Gain)

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_p(s)\, G_f(s)\, G_c(s)}{1 + G_p(s)\, G_f(s)\, G_c(s)\, G_m(s)}$$

$1 + G_p(s)\, G_f(s)\, G_c(s)\, G_m(s) = 0$ [Characteristic Equation]

From Figure 1: A feedback control system

i. $G_p(s) = \frac{1}{\tau_p s + 1}$

but ~~and~~ $\tau_p = 1$

$$G_p(s) = \frac{1}{s+1}$$

ii. $G_f(s) = \frac{1}{\tau_f s + 1}$ but $\tau_f = 1/2$

$$G_f(s) = \frac{1}{1/2\, s + 1}$$

$$G_f(s) = \frac{\frac{1}{1/2}}{\frac{1/2}{1/2} s + \frac{1}{1/2}}$$

$$G_f(s) = \frac{2}{s+2}$$

iii) $G_c(s) = k_c(s)$

iv) $G_m(s) = \frac{1}{\tau_m s+1}$ but $\tau_m = 1/3$

$$G_m(s) = \frac{1}{1/3\, s+1}$$

$$G_m(s) = \frac{3}{s+3}$$

Substituting $G_p(s)$, $G_f(s)$, $G_c(s)$ and $G_m(s)$ into the characteristic, it gives:

$$1 + \frac{1}{s+1} \cdot \frac{2}{s+2} \cdot k_c \cdot \frac{3}{s+3} = 0$$

$$1 + \frac{6k_c}{(s+1)(s+2)(s+3)} = 0 \longrightarrow \frac{1}{1} + \frac{6k_c}{(s+1)(s+2)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s+3) + 6k_c}{(1)\ (s+1)(s+2)(s+3)} = 0$$

Cross Multiplying

$(s+1)(s+2)(s+3) + 6k_c = 0$ opening the brackets

$(s^2+2s+s+2)(s+3) + 6k_c = 0$

$(s^2+3s+2) \times (s+3) + 6k_c = 0$ opening the brackets

$s^3 + 3s^2 + 2s + 3s^2 + 9s + 6 + 6k_c = 0$

$s^3 + 6s^2 + 11s + 6 + 6k_c = 0$

Applying Routh Criterium, we have the general formula to be:

Row 1 : a_0 a_2 a_4 a_6 - - - - - - - -

Row 2 : a_1 a_3 a_5 a_7 - - - - - - - -

Row 3 : A_1 A_2 A_3 - - - - - - -

Row 4 : B_1 B_2 B_3 - - - - - -

Row 5 : C_1 C_2 C_3 - - - - - -

Hence, $a_0 = 1$, $a_1 = 6$

$a_2 = 11$, $a_3 = 6 + 6k_c$

Substituting the coefficients into the Routh criterium, we have:

Row 1 : 1 11

Row 2 : 6 $6+6k_c$

Row 3 : $10-k_c$ 0

Row 4 : $6+6k_c$ 0

$$A_1 = \frac{(6\times 11) - (1\times[6+6k_c])}{6}$$

$$A_1 = \frac{66 - [6+6k_c]}{6}$$

$$A_1 = \frac{60 - 6k_c}{6}$$

$A_1 = 10 - k_c$ $A_2 = 0$ $A_3 = 0$

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$$B_1 = \frac{(10-k_c)(6+6k_c) - (6)(0)}{10-k_c}$$

$B_1 = 6 + 6k_c$ $\quad B_2 = 0 \quad C_1 = 0$ ----

Hence, all the values in the first column must be positive and greater than 0 for the control system to be stable using Routh's criterium

From 'Row 3'

$$10 - k_c > 0$$
$$-k_c > -10$$
$$k_c < \frac{-10}{-1} = k_c < 10$$

From 'Row 4'

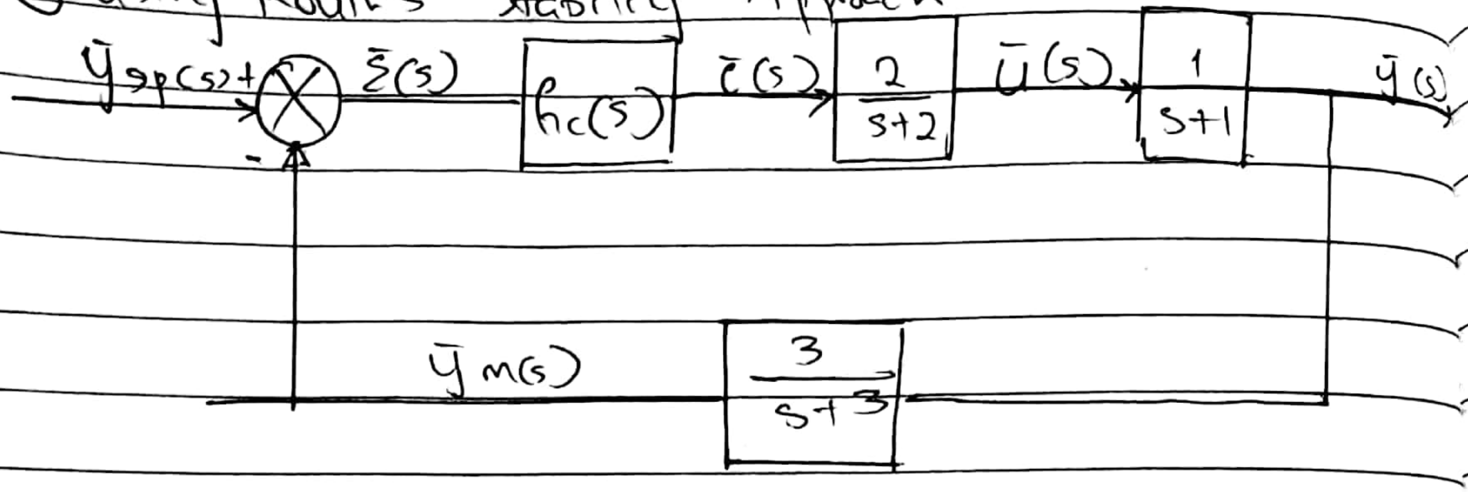
$$6 + 6k_c > 0$$
$$6k_c > -6$$
$$k_c > \frac{-6}{6} = k_c > -1$$

Hence $k_c < 10$ and $k_c > -1$

Therefore the range of values of k_c for which the system is stable is $-1 < k_c < 10$

$\therefore k_c = [0 \; 1 \; 2 \; 3 \; 4 \; 5 \; 6 \; 7 \; 8 \; 9]$ Natural number.

② Using Routh's Stability Approach



Where

$$y(s) = \frac{G_p(s)\,G_f(s)\,G_c(s)}{1 + G_p(s)\,G_f(s)\,G_c(s)\,G_m(s)} \cdot y_{sp}(s)$$

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_p(s)\,G_f(s)\,G_c(s)}{1 + G_p(s)\,G_f(s)\,G_c(s)\,G_m(s)}$$

And

$1 + G_p(s) G_f(s) G_c(s) G_m(s) = 0$ [Characteristic Equation]

Using a PI controller, $G_c = k_c + \frac{k_c}{\tau_I s}$

$k_c = 5 \qquad \tau_I = 0.25$

$$G_c = \frac{5}{1} + \frac{5}{0.25s}$$

$$G_c = \frac{1.25s + 5}{0.25s}$$

$$G_c = \frac{\frac{1.25s}{0.25} + \frac{5}{0.25}}{\frac{0.25s}{0.25}}$$

$$G_c = \frac{5s+120}{s}$$

Substituting G_p, G_f, G_c and G_m into the characteristic equation, we have:

$$1 + \frac{1}{s+1} \cdot \frac{2}{s+2} \cdot \frac{5s+120}{s} \cdot \frac{3}{s+3} = 0$$

$$1 + \frac{6(5s+120)}{(s+1)(s+2)(s)(s+3)} = 0$$

$$\frac{1}{1} + \frac{30s+120}{(s+1)(s+2)(s)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s)(s+3) + 30s+120}{(s+1)(s+2)(s)(s+3)} = 0$$

Cross Multiplying

$$(s+1)(s+2)(s)(s+3) + 30s + 120 = 0$$

$$(s)(s+1)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^2+s)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^3+2s^2+s^2+2s)(s+3) + 30s + 120 = 0$$

$$s^4 + 2s^3 + s^3 + 2s^2 + 3s^3 + 6s^2 + 3s^2 + 6s + 30s + 120 = 0$$

$$s^4 + 6s^3 + 11s^2 + 36s + 120 = 0$$

Using Routh criterion, we have

Row 1:	1	11	120
Row 2:	6	36	0
Row 3:	5	120	0
Row 4:	-108	0	
Row 5:	120		

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$$A_1 = \frac{(6)(11) - (1)(36)}{6}$$

$$= \frac{66 - 36}{6} = \underline{5}$$

$$A_2 = \frac{(6)(120) - (1)(0)}{6}$$

$$= \frac{720 - 0}{6} = \underline{120}$$

$A_3 = 0$

$$B_1 = \frac{(5)(36) - (6)(120)}{5}$$

$$= \frac{180 - 720}{5} = -108$$

$B_2 = 0$

$B_3 = 0$

$$C_1 = \frac{(-108)(120) - (5)(0)}{-108}$$

$$= \underline{120}$$

$C_2 = 0,$ $\quad C_3 = 0$

The system is UNSTABLE because not all the values in the first column of the Routh criterion is positive.