15/ENG03/007
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ENG 381 Assignment 4

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$\div y = (1-x^2)y''$

$V = (1-x^2)$ $V' = -2x$ $V'' = -2$ $V''' = 0$

$U = y''$ $U^n = y^{n+2}$

$y^n = U^nV + nU^{(n-1)}V^{(1)} + \frac{n(n-1)}{2!}U^{(n-2)}V^{(2)} + \ldots$

$= y^{n+2}\cdot(1-x^2) + (n+2)y^{(n+1)}\cdot(-2x) + \frac{(n+2)(n+1)}{2!}y^n\cdot(-2)$

$= (1-x^2)y^{n+2} - 2x(n+2)y^{(n+1)} + \frac{(-2)(n+2)(n+1)}{2!}y^n$

$\div y = -2xy'$

$V = -2x$ $V' = -2$ $V'' = 0$

$U = y'$ $U^n = y^{n+1}$

$y^n = y^{(n+1)}(-2x) + (n+1)y^n\cdot(-2)$

$= -2xy^{(n+1)} - 2(n+1)y^n$

$\div y = 2y$

$V = 2$ $V' = 0$

$U = y$ $U^n = y^n$

$y^n = 2y^n$

$\therefore y^n = [(1-x^2)y^{n+2} - 2x(n+2)y^{n+1} - \frac{2(n+2)(n+1)}{2}y^n - 2xy^{n+1}$

$- 2(n+1)y^n + 2y^n] = 0$

at $x = 0$

$0 = y^{n+2} - [n^2-n]y^n - 2ny^n + 2y^n$

$y^{n+2} = [n^2 - n + 2n - 2]y^n$

$y^{n+2} = [n^2 + n - 2]y^n$

$n = 1, 2, 3, 4, \ldots$

$n = 1$

$y^3 = y''' = [1^2 - 1 + 2 - 2] y' = 0$

$n = 2$

$y^4 = y^{iv} = [2^2 + 2 - 2] y'' = 4y''$

$n = 3$

$y^5 = y^{v} = [3^2 + 3 - 2] y''' = 10y''' = 0$

$n = 4$

$y^6 = y^{vi} = [4^2 + 4 - 2] y^{iv} = 18y^{iv} = 18y^{iv} = 18[4y'']$

$n = 5$

$y^7 = y^{vii} = [5^2 + 5 - 2] y^{v} = 28y^{v} = 0$

$n = 6$

$y^8 = y^{viii} = [6^2 + 6 - 2] y^{vi} = 40y^{vi} = [40 \times 18 \times 4] y''$

$$y = 1 + y^0 + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y''(4)}{6!} + \frac{40[18 \times 4] y''}{8!}$$

$$y = 1 + y^0 + y' + y''\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14}\right]$$

Number. 2.

2(i) $L[3e^{-4t} - 5e^{4t}] = 3L[e^{-4t}] - 5L[e^{4t}]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$ //

(ii) $L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

(iii) $L[t^3 + 2t^2 - t + 4] = L[t^3] + 2L[t^2] - L[t] + L[4]$

$= \frac{3}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$ //

(iv) $L[e^{-2t} \cos 5t]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t} \cos 5t] = \frac{(s+2)}{(s+2)^2+25}$ //

(v) $L[t \sin 3t]$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t \sin 3t] = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

$u = 3 \qquad V = s^2+9$

$du = 0 \qquad dv = 2s$

$-\frac{d}{ds}\left[\frac{3}{s^2+9}\right] = -\left[\frac{Vdu - Udv}{V^2}\right] = -\left[\frac{0 - 3(2s)}{(s^2+9)^2}\right] = \frac{+6s}{(s^2+9)^2}$ //

(vi) $L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \cancel{\int_{s=s}^{\alpha} \frac{1}{s+}} \int_{s=s}^{\alpha} F(s)\, ds$

$$= \int_s^\infty \left(\frac{1}{s+1} - \frac{1}{s+2}\right) ds$$

$$= \int_s^\infty \frac{1}{s+1} ds - \int_s^\infty \frac{1}{s+2} ds$$

$$= \Big[\ln(s+1) - \ln(s+2)\Big]_s^\infty$$

$$= \ln\left[\frac{s+1}{s+2}\right]\Bigg|_s^\infty$$

$$= 0 - \ln\frac{s+1}{s+2}$$

(vii) $L[e^{4t}\cos 2t]$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{(s-4)}{(s-4)^2+4}$$

(viii) $L[t\sin 2t]$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\sin 2t] = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$$

$u = 2$ $\quad\quad$ $v = s^2+4$

$du = 0$ $\quad\quad$ $dv = 2s$

$$-\frac{d}{ds}\left[\frac{2}{s^2+4}\right] = -\left[\frac{vdu - udv}{v^2}\right] = -\left[\frac{0 - 4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$$

(ix) $L[t^2 + 4t^2 + 5] = L[t^2] + L[4t^2] + L[5]$

$$= \frac{2}{s^3} + \frac{8}{s^3} + \frac{5}{s}$$

(x) $L[t^2\cos t]$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$L[t^2\cos t] = -\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$u = s \qquad du = 1$

$v = s^2+1 \qquad dv = 2s$

$\Rightarrow \dfrac{v\,du - u\,dv}{v^2} = \dfrac{(s^2+1) - 2s^2}{s^4+2s^2+1} = \boxed{\dfrac{(s^2+1) - 2s^2}{(s^2+1)(s^2+1)}}$

$u = (s^2+1) - 2s^2 \qquad du = 2s - 4s = -2s$

$v = (s^2+1)(s^2+1) \qquad dv = 4s^3 + 4s$

$\therefore -\dfrac{d^2}{ds^2}\left[\dfrac{s}{s^2+1}\right] = -\dfrac{(s^2+1)^2(-2s) - (1-s^2)(4s^3+4s)}{(s^4+2s^2+1)^2}$

$= -\dfrac{(s^4+2s^2+1)(-2s) - (1-s^2)(4s^3+4s)}{(s^4+2s^2+1)^2}$

$= -\dfrac{(2s^5 - 4s^4 - 6s)}{(s^4+2s^2+1)^2}$

(xii) $L\left[\dfrac{\sinh 2t}{t}\right]$

$L[\sinh 2t] = \dfrac{2}{s^2-4}$

$L\left[\dfrac{\sinh 2t}{t}\right] = \int_{s=s}^{\infty} f(s)\,ds = \int_s^{\infty} \dfrac{2}{s^2-4}\,ds$

$= 2\int_s^{\infty} \dfrac{1}{s^2-4}\,ds = 2\ln(s^2-4)$

(x) $L[e^{3t}(t^2+4)]$

$L[(t^2+4)] = \dfrac{2}{s^3} + \dfrac{4}{s}$

$L[e(t^2+4)] = \dfrac{2}{(s-3)^2} + \dfrac{4}{(s-3)}$

Number. 8.

(i) $\dfrac{s-5}{(s-3)(s-4)} \Rightarrow \dfrac{A}{(s-3)} + \dfrac{B}{(s-4)} \Rightarrow \dfrac{A(s-4)+B(s-3)}{(s-3)(s-4)}$

$\Rightarrow s-5 = A(s-4) + B(s-3)$ — (i)

at $s-3=0$, $s=3$ substitute in (i)

$3-5 = A(3-4) + 0$

$-2 = -A$

$A = 2$

at $s-4=0,\ s=4$

$4-5 = 0 + B(4-3)$

$-1 = B$

$B=-1$

$$\Rightarrow L^{-1}\left[\frac{2}{(s-3)} + \frac{(-1)}{(s-4)}\right]$$

$$= 2e^{3t} - e^{4t}$$

(ii) $$\frac{2s-6}{(s-2)(s-4)} \Rightarrow \frac{A}{(s-2)} + \frac{B}{(s-4)} \Rightarrow \frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$$

$2s-6 = A(s-4)+B(s-2)$

at $s-4=0,\ s=4$

$2(4)-6 = 0 + B(4-2)$

$2 = 2B$

$\boxed{B=1}$

at $s-2=0,\ s=2$

$2(2)-6 = A(2-4)+0$

$-2 = -2A$

$A=1$

$$L^{-1}\left[\frac{1}{(s-2)} + \frac{1}{(s-4)}\right]$$

$$= e^{2t} + e^{4t}$$

(iii) $$\frac{5s-8}{s(s-4)} \Rightarrow \frac{A}{s} + \frac{B}{s-4} \Rightarrow \frac{A(s-4)+Bs}{s(s-4)}$$

$5s-8 = A(s-4)+Bs$

at $s=0$

$0-8 = A(-4)$

$A=2$

at $s-4=0\ \ s=4$

$5(4)-8 = B(4)$

$12 = 4B$

$\cancel{B=4}$ $B=3$

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$$L^{-1}\left[\frac{2}{s} + \frac{3}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

(W) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} \Rightarrow \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$\frac{A(s-1)(s-1)^2 + B(s-3)(s-1)^2 + C(s-3)(s-1)}{(s-3)(s-1)(s-1)^2} = \frac{s^2-3s-4}{(s-3)(s-1)^2}$$

$$\cancel{s^2-3s+4 = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{}}$$

$$s^2-3s-4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

$$3^2-3(3)-4 = A(3-1)^2 \qquad \Rightarrow A = -1$$

$$1^2-3(1)-4 = C(1-3) \qquad \Rightarrow C = 3$$

$$s^2-3s-4 = [s^2-2s+1)A + (s^2-4s+3)B + (s-3)C$$

$$= -2A - 4B + C = -3$$

$$-2(-1) - 4(B) + 3 = -3$$

$$-4B = -3-3-2$$

$$B = 2$$

$$L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right]$$

$$= -e^{3t} + 2e^{t} + 3!e^{t}$$

(V) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16} = \frac{s-5}{(s+2)^2+4}$$

$$\frac{s-5}{(s+2)^2+4} \Rightarrow (e^{2t}-7)\cos 4t$$