UNDIE FIDELIS BEWHAPUYE
15/ENG03/033
CIVIL ENGINEERING
ENG 381

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

General equation

$$W^n = U^n V^0 + n x^{n-1} V' + \frac{n(n-1)U^{n-2}V''}{2!} + \frac{n(n-1)(n-2)U^{n-3}V'''}{3!} +$$

for $(1-x^2)y''$ let $u = y''$, $u' = y'''$, $U^n = y^{n+2}$

$V = V^0 = 1-x^2$, $V' = -2x$, $V'' = -2$, $V''' = 0$

for xy'

$u = y'$, $u = y''$, $u'' = y'''$, $U^n = y^{n+1}$

$V = V^0 = x$ $\quad V' = 1$, $V'' = 0$

$W_2^n = x \cdot y^{n+1} + n \cdot y^n \cdot 1$

$= xy^{n+1} + ny^n$

for $y = y^n = W_3^n$

$W^n = W_1^n + W_2^n + W_3^n$

$W^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2(xy^{n+1} + ny^n)$

$0 = (1-x^2)y^{n+2} - 2xny^{n+1} - (n^2-n)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

If $x = 0$

$0 = y^{n+2} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^{n+2} = (n^2 - n + 2n - 2)y^n$

$y^{n+2} = (n^2 + n - 2)y^n$

for $n = 1$

$y^3 = y''' = (1^2 - 1 + 2 - 2)y^1 = 0$

for $n = 2$

$y^4 = y^{IV} = (2^2 + 2 - 2)y'' = 4y''$

for $n = 3$

$y^5 = y^V = (3^2 + 3 - 2)y''' = 10y''' = 0$

for $n = 4$

$y^6 = y^{VI} = (4^2 + 4 - 2)y^{IV} = 18y^{IV} = 18(4y'')$

for $n = 5$

$$y^n = y^{vii} = (5^2+5-2)y^v = 28y^v = 0$$

for $n = 8$

$$y^8 = y^{viii} = (6^2+6-2)y^{vi} = 40y^{vi} = (40 \times 18 \times 4)y''$$

$$y = 1 + f(x) + y'f'(x) + \frac{y''f''(x)}{2!} +$$

$$y = 1 + y^0 + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y''(4)}{6!} + \frac{40(18\times4)y''}{8!}$$

$$y = 1 + y^0 + y' + y''\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14}\right)$$

2) $$L(3e^{-4t} - 5e^{4t}) = \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $$L(\sin 4t + \cos 4t) = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$$

iii) $$(t^3 + 2t^2 - t + 4) = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$$

$$= \frac{1}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv) $$(e^{-2t}\cos 5t) = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+4s+29}$$

v) $$(t\sin 3t) = (-1)^1 \frac{d^n}{dx^n}(f(t)) = -1\frac{d}{dx}\left(1, \frac{3}{s^2+3^2}\right)$$ $du = 0$ $dv = 25$

$$= -\left(\frac{-6s}{(s^2+9)^2}\right) = \frac{6s}{(s^2+9)^2}$$

vi) $$\left(\frac{e^{-t} - e^{-2t}}{t}\right) = \left(\frac{\frac{1}{s+1} - \frac{1}{s+2}}{\frac{1}{s^{1+1}}}\right) = \left(\frac{1}{s+1} - \frac{1}{s+2}\right) \cdot s^2$$

$$= \frac{s^2}{(s+1)(s+2)}$$

vii) $\left(e^{4t}\cos 2t\right) = \frac{s}{(s-4)^2+2^2} = \frac{s}{s^2-8s+20}$

viii) $(t\sin 2t) = -1\cdot\frac{d}{dx}\left(\frac{2}{s^2+2^2}\right)$ $du=0$, $dv=2s$

$= -1\cdot\left(\frac{-4s}{(s^2+2^2)^2}\right)$ $du=0$, $dv=2s$ $= \frac{4s}{(s^2+4)^2}$

ix) $t^3+4t^2+5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2+4) = t^2e^{3t} + 4e^{3t} = \frac{2!}{(s-3)^{1+1}} + \frac{4}{s-3}$

$= \frac{2}{(s-3)^2} + \frac{4}{s-3}$

xi $t^2\cos t = (-1)^2\cdot\frac{d^2}{dx^2}\left(\frac{s}{s^2+1}\right) = \frac{d}{dx}\left[\frac{d}{dx}\left(\frac{s}{s^2+1}\right)\right]$ $du=1$, $dv=2s$

$\frac{d}{dx}\left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right) = \frac{d}{dx}\left(\frac{1-s^2}{(s^2+1)^2}\right)$ $du=-2s$, $dv=4s^3+4s$

$=\left(\frac{-2s^5-4s^3-2s-4s+4s^5}{\left((s^2+1)^2\right)^2}\right) = \left(\frac{2s^5-4s^3-6s}{(s^2+1)^4}\right)$

x) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-4)-\ln s$

3i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

$3-5 = A(3-4) \Rightarrow A=2$

$4-5 = B(4-3) \Rightarrow B=-1$

$L^{-1}\left\{\frac{2}{s-3} - \frac{1}{s-4}\right\} = 2e^{3t} - e^{4t}$

ii) $\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{s-2} + \dfrac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2(2)-6 = A(s-4) \Rightarrow A=1$

$2(4)-6 = B(4-2) \Rightarrow B=1$

$L^{-1}\left(\dfrac{1}{s-2} + \dfrac{1}{s-4}\right) = e^{2t} + e^{4t}$

iii) $\dfrac{5s-8}{s(s-4)} = \dfrac{A}{s} + \dfrac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

$5(0)-8 = A(0-4) \Rightarrow A=2$

$5(4)-8 = B(4) \Rightarrow B=3$

$L^{-1}\left(\dfrac{2}{s} + \dfrac{3}{s-4}\right) = 2u(t) + 3e^{4t}$

iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{s-1} + \dfrac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-1)^2 + B(s-3)(s-1) + C(s-3)$

$3^2-3(3)-4 = A(3-1)^2 \quad A \Rightarrow = -1$

$1^2-3(1)-4 = C(1-3) \Rightarrow C=3$

$s^2-3s-4 = (s^2-2s+1)A + (s^2-4s+3)B + (s-3)C$

$= -2A-4B+C = -3$

$-2(-1) - 4(B) + 3 = -3$

$-4B = -3-3-2$

$B = 2$

$L^{-1}\left(\dfrac{-1}{s-3} + \dfrac{2}{s-1} + \dfrac{3}{(s-1)^2}\right)$

$= -e^{3t} + 2e^{t} + 3 | e^{t}$

v) $\dfrac{s-5}{s^2+4s+20}$

$$= \frac{s-5}{s^2+4s+4+16}$$

$$= \frac{s-5}{(s+2)^2+4}$$

$$= \frac{s-5}{(s+2)^2+4} \quad = (e^{2t}-7)\cos 4t$$