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Mechanical Engineering

1 $(1-x^2)\dfrac{d^2y}{dx} - 2x\dfrac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y^{(0)} = 0$

let $u^0 = y''$, $u^1 = y'''$, $u'' = y''''$

$v^0 = (1-x^2)$ $v' = 2x$ $v'' = -2$

$W' = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \dfrac{(-2)(n)(n-1)}{2!}y^n$

$W_2 = -(y^{n+1}(2x) + ny^{(n)}(2))$

$y^{(n+2)}(1\cancel{-}x^2) + 2xny^{(n+1)} - n(n-1)y^n - 2xy^{(n+1)} + 2ny^{(n)} + 2y^{(n)} = 0$

$0 = y^{(n+2)}(1-x^2) + y^{(n+1)}(2xn - 2x) + y^{(n)}(-2n+2-n(n-1))$

when $x = 0$

$y^{(n+2)}(1-0) + y^{(n+1)}(0-0) + y^{(n)}(-n^2-n+2) = 0$

$y^{(n+2)} + y^{(n)}(-n^2-n+2) = 0$

$y^{(n+2)} = y^{(n)}(n^2+n-2) = 0$ recurrent solution.

when $n=0$ $y^{(0+2)} = y^0(0+0-2) = y^0_{(0)}(-2) = -2y^0_{(0)}$

$n=1$ $y^{(3)}_{(0)} = y'(1^2+1-2) = y'_{(0)}(0) = 0$

$n=2$ $y^{(4)}_{(0)} = y^2_{(0)}(2^2+2-2) = y^2_{(0)}(4) = 4\times -2y^0_{(0)}$

$n=3$ $y^{(5)}_{(0)} = y^3_{(0)}(3^2+3-2) = y^3_{(0)}(10) = 10\times 0$

$n=4$ $y^{(6)}_{(0)} = y^{(4)}_{(0)}(4^2+4-2) = y^4_{(0)}(18) = 18\times 4\times -2y^0$

$n=5$ $y^7_{(0)} = y^{(5)}_{(0)}(5^2+5-2) = y^5_{(0)}(28) = 28\times 10\times 0$

$n=6$ $y^8_{(0)} = y^{(6)}_{(0)}(6^2+6-2) = y^6_{(0)}(40) = 40\times 18\times 4\times -2y^0_{(0)}$

$y = y^0_{(0)} + xy'_{(0)} + \dfrac{x^2}{2!}\cdot y^2_{(0)} + \dfrac{x^3}{3!}\cdot y^3_{(0)} + \dfrac{x^4}{4!}y^4_{(0)} + \dfrac{x^5}{5!}y^5_{(0)}$

$+ \dfrac{x^6}{6!}y^6_{(0)} + \dfrac{x^7}{7!}y^7_{(0)} + \dfrac{x^8}{8!}y^8$

$$y = y^0_{(0)} + x y'_{(0)} + \frac{x^2}{2\times 1}(-2y^0) + \frac{x^4}{4\times 3\times 2\times 1}(4\times -2y^0) + \frac{x^6}{6\times 5\times 4\times 3\times 2\times 1}(18\times 4\times -2y^0) + \frac{x^8(40\times 18\times 4\times -2y^0)}{8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}$$

$$y = y^0_{(0)}\left(1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} + \cdots\right) + x y'(0)$$

i.) $f(t) = 3e^{-4t} - 5e^{4t}$

$$F(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

ii.) $\sin 4t + \cos 4t = f(t)$

$$F(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

iii.) $f(t) = t^3 + 2t^2 - t + 4$

$$F(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv.) $e^{-2t}\cos 5t = \dfrac{s}{(s+2)^2+25}$

v.) $t\sin 3t = -\dfrac{d}{ds}\left(\dfrac{3}{s^2+1}\right) = \dfrac{[(s^2+1)(0) - (3)(2s)]}{(s^2+1)^2}$

$$= \frac{6s}{(s^2+1)^2}$$

i.) $f(t) = \dfrac{e^{-t} - e^{-2t}}{t}$

$F(s) = \int_0^{\infty} \frac{1}{6+1} d6 - \int_0^{\infty} \frac{1}{6+2} d6$

$= \left(\ln(6+1) - \ln(6+2)\right)_s^{\infty}$

$= \left(\ln\left(\frac{6+1}{6+2}\right)\right)_s^{\infty}$

$= \left(-\ln\left(\frac{s+1}{s+2}\right) + \left(\ln\left(\frac{\infty}{\infty}\right)\right)\right)$

$= \ln(s+2) - \ln(s+1)$

G.S. = P.I. + C.F.

y = P.I.

let $6^2 + 4 = u$ $\quad \frac{du}{d6} = 26$; $d6 = \frac{du}{26}$

$\int_0^{\infty} \frac{2}{u} \frac{du}{26} = \frac{2}{26} \ln u = \frac{1}{6} \ln u$

vii.) $e^{4t}\cos 2t = \dfrac{s}{(s-4)^2 + 4}$

viii.) $f(t) = t\sin 2t$

$F(s) = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{(s^2+4)(0) - (2)(2s)}{(s^2+4)^2}$

$= \frac{4s}{(s^2+4)^2}$

ix.) $f(t) = t^3 + 4t^2 + 5$

$F(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x.) $f(t) = e^{3t}(t^2+4) = t^2e^{3t} + 4e^{3t}$

$F(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

xi.) $f(t) = t^2\cos t$

$F(s) = (-1)^2 \frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right) = \frac{(s^2+1)(1) - (s)(2s)}{(s^2+1)^2}$

$= \frac{d}{ds}\left(\frac{-s^2+1}{(s^2+1)^2}\right) = \frac{(s^2+1)^2(-2s) - [(-s^2+1)(}{(s^2+1)^4}$

xii.) $\dfrac{\sinh 2t}{t}$

$F(s) = \int_6^{\infty} \frac{2}{6^2+4} d6$ $\quad$ let $u = 6^2+4$ $\quad \frac{du}{d6} = 26$ $\quad d6 = \frac{du}{26}$

$= \frac{1}{26}\int_6^{\infty} \frac{2}{u} d6 = \left[\frac{\ln u}{6}\right]_s^{\infty}$

$= \left(\frac{\ln(6^2+4)}{6}\right)_s^{\infty}$

$= \frac{\ln(\infty^2+4)}{4} - \frac{\ln(s^2+4)}{6}$

3i.) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s = 3 \Rightarrow 3 - 5 = A(3-4) + 0$

$-2 = -A$; $A = 2$

$s = 4$; $4 - 5 = B(4-3)$

$-1 = B$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4} = 2e^{3t} - e^{4t}$

$$) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s=2;\ 2(2)-6 = A(2-4)$

$-2 = -2A;\ A=1$

$s=4;\ 2(4)-6 = B(4-2)$

$2 = 2B;\ B=1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$$

$$iii)\ \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

f $s=0;\ 5(0)-8 = A(0-4)$

$-8 = -4A;\ A=2$

f $s=4;\ 5(4)-8 = B(4)$

$12 = 4B;\ B=3$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$$

$$3(iv)\ f(t) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)^2}$$

$s^2-3s-4 = A(s-1)^2 + B(s-3) + C(s-3)(s-1)$

$s^2-3s-4 = As^2 - 2As + A + Bs - 3B + Cs^2 - 4Cs + C$

$s^2-3s-4 = s^2(A+C) + s(-2A+B-4C) + A - 3B + C$

$A+C=1$

$-2A+B-4C=-3$

$A-3B+C=-4$

$A=\frac{-1}{3},\ B=\frac{5}{3},\ C=\frac{4}{3}$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{3(s-3)} + \frac{5}{3(s-1)^2} + \frac{4}{3(s-1)}$$

$$= \frac{1}{3}\left(-e^{3t} + 5te^{t} + 4e^{t}\right)$$

$$v)\ \frac{s-5}{s^2+4s+20} = \frac{Ax+B}{s^2+4s+20}$$

$s-5 = Ax + B(s^2+4s+20)$

$Ax + B = 0 \qquad B = -Ax$

$4Ax + 4B = 1 \qquad 4Ax + 4(-Ax) = 1$

$20Ax + 20B = -5$