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1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

Soln

$(1-x^2)y'' - 2xy' + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y^{(0)} = 0$

$(1-x^2)y^{(n+2)} + n \cdot y^{(n+1)} \cdot (-2x) + \frac{n(n-1)y^{(n)} \cdot (\cancel{-2})}{\cancel{2}!} - 2x \cdot y^{(n+1)} - n(2)y^{(n)} + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - (2n+2)xy^{(n+1)} - n(n-1) - 2n + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - 2x(n+1)y^{(n+1)} - n^2 + n - 2n + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - 2x(n+1)y^{(n+1)} - n^2 - n + 2y^{(n)} = 0$

$(1-x^2)y^{(n+2)} - 2x(n+1)y^{(n+1)} - (n^2+n-2)y^{(n)} = 0$

let $x = 0$

$(y^{(n+2)})_0 = (n^2+n-2)(y^{(n)})_0$

when $n = 0$

$(y^{(2)})_0 = -2(y^{(0)})_0$

when $n = 1$

$(y^{(3)})_0 = 0$

when $n = 2$

$(y^{(4)})_0 = 4(y^{(2)})_0 = (4)(-2)(y^{(0)})_0$

when $n = 3$

$(y^{(5)})_0 = 10(y^{(3)})_0 = 10(0) = 0$

when $n = 4$

$(y^{(6)})_0 = 18(y^{(4)})_0 = 18(4)(y^{(0)})_0(-2)$

when $n = 5$

$(y^{(7)})_0 = 28(y^{(5)})_0 = 28(0) = 0$

when $n = 6$

$(y^{(8)})_0 = 40(y^{(6)})_0 = 40(18)(4)(y^{(0)})_0(-2)$

from Maclaurin's series

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0 + \frac{x^7}{7!}(y^{(7)})_0 + \cdots$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{\cancel{2!}}(\cancel{-2})(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(y^{(0)})_0(-2) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \cdots$

$\cdots + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(40)(18)(4)(-2)(y^{(0)})_0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 - \frac{x^4}{3}(y^{(0)})_0 - \frac{x^6}{5}(y^{(0)})_0 - \frac{x^8}{7}(y^{(0)})_0$

$y = (y^{(0)})_0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right] + x(y^{(1)})_0$

2 i $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} = \frac{3}{s+4} - \frac{5}{s-4}$

ii $L\{\sin 4t + \cos 4t\} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii $L\{t^3 + 2t^2 - t + 4\} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv $L\{e^{-2t}\cos 5t\} = \frac{s}{(s+2)^2 + 25}$

v $L\{t\sin 3t\} = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right) = \frac{6s}{(s^2+9)^2}$

$L\{\sin 3t\} = \frac{3}{s^2+9}$

Using quotient rule, $\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\}$

$\lim_{t\to 0}\left(\frac{e^{-t} - e^{-2t}}{t}\right) = \frac{1-1}{0} = \frac{0}{0}$ undefined

$\frac{-e^{-t} + 2e^{-2t}}{+1} = \frac{-1+2}{+1} = \frac{1}{+1} = +1$

$L\{e^{-t} - e^{-2t}\} = \frac{1}{s+1} - \frac{1}{s+2} = f(\sigma)$

$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty}\left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right)d\sigma$

$= \int_{\sigma=s}^{\infty}\frac{1}{\sigma+1}\,d\sigma - \int_{\sigma=s}^{\infty}\frac{1}{\sigma+2}\,d\sigma$

$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_{\sigma=s}^{\infty}$

$= \left[\ln\left(\frac{\sigma+1}{\sigma+2}\right)\right]_{\sigma=s}^{\infty}$

$= \ln\frac{\infty+1}{\infty+2} - \ln\frac{s+1}{s+2}$

$= \ln 1 - \ln\frac{s+1}{s+2}$

$= 0 - \ln\frac{s+1}{s+2}$

$= -\ln\frac{s+1}{s+2}$

$= \ln\frac{s+2}{s+1}$

vii $L\{e^{4t}\cos 2t\} = \dfrac{s-4}{(s-4)^2+4}$

viii $L\{\sin 2t\} = \dfrac{2}{s^2+4}$

$L\{t\sin 2t\} = \dfrac{(s^2+4)0 - 2(2s)}{(s^2+4)^2} = \left(\dfrac{-4s}{(s^2+4)^2}\right) \cdot -\dfrac{d}{ds} = \dfrac{4s}{(s^2+4)^2}$

ix $L\{t^3+4t^2+5\} = \dfrac{6}{s^4} + \dfrac{8}{t^3} + \dfrac{5}{s}$

x $e^{3t}(t^2+4) = e^{3t}\cdot t^2 + e^{3t}\cdot 4 = t^2e^{3t} + 4e^{3t}$

$L\{t^2e^{3t} + 4e^{3t}\} = \dfrac{2}{(s-3)^3} + \dfrac{4}{s-3}$

xi $t^2\cos t$

$L\{\cos t\} = \dfrac{s}{s^2+1}$

$L\{t^2\cos t\} = +\dfrac{d^2}{ds^2}\left(\dfrac{s}{s^2+1}\right)$

$\dfrac{d}{ds}\left(\dfrac{s}{s^2+1}\right) = \dfrac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} = \dfrac{s^2+1-2s^2}{(s^2+1)^2} = \dfrac{-s^2+1}{(s^2+1)^2}$

$L\{t\cos t\} = -\dfrac{d}{ds}\left(\dfrac{s}{s^2+1}\right) = -\left(\dfrac{-s^2+1}{(s^2+1)^2}\right) = \dfrac{s^2-1}{(s^2+1)^2}$

$\dfrac{d}{ds}\left(\dfrac{s^2-1}{(s^2+1)^2}\right) = \dfrac{(s^2+1)^{\cancel{2}}(2s) - (s^2-1)(2\cancel{(s^2+1)}\cdot 2s)}{(s^2+1)^{\cancel{4}\,3}} = \dfrac{2s(s^2+1) - 4s(s^2-1)}{(s^2+1)^3}$

$= \dfrac{2s[(s^2+1) - 2(s^2-1)]}{(s^2+1)^3}$

$= \dfrac{2s(s^2+1-2s^2+2)}{(s^2+1)^3}$

$= \dfrac{2s(-s^2+3)}{(s^2+1)^3}$

$L\{t^2\cos t\} = -\left(\dfrac{2s(-s^2+3)}{(s^2+1)^3}\right) = \dfrac{2s(s^2-3)}{(s^2+1)^3}$

xii $\sinh 2t/t$

$\lim_{t\to 0}\left(\dfrac{\sinh 2t}{t}\right) = \dfrac{\sinh 0}{0} = \dfrac{0}{0}$ undefined

$\lim_{t\to 0}\left(\dfrac{2\cosh 2t}{1}\right) = \dfrac{2\cosh 0}{1} = 2$

$$L\{\sinh 2t\} = \frac{2}{s^2+4} = f(\sigma)$$

$$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2+4}\, d\sigma$$

$$= \cancel{2}\left[\frac{1}{\cancel{2}} \tan^{-1}\frac{\sigma}{2}\right]_{\sigma=s}^{\infty}$$

$$= \left[\tan^{-1}\frac{\sigma}{2}\right]_{\sigma=s}^{\infty}$$

$$= \tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$$

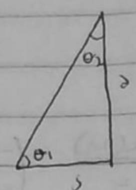
$$= \tan^{-1}\infty - \tan^{-1}\frac{s}{2}$$

$$= 90^\circ - \tan^{-1}\frac{s}{2}$$

$$= \frac{\pi}{2} - \tan^{-1}\frac{s}{2}$$

$$= \tan^{-1}\frac{2}{s} + \cancel{\tan^{-1}\frac{s}{2}} - \cancel{\tan^{-1}\frac{s}{2}}$$

$$= \tan^{-1}\frac{2}{s}$$



$\tan\theta_1 = \frac{2}{s}$

$\theta_1 = \tan^{-1}\frac{2}{s}$

$\tan\theta_2 = \frac{s}{2}$

$\theta_2 = \tan^{-1}\frac{s}{2}$

$\theta_1 + \theta_2 = 90^\circ = \frac{\pi}{2}$

$\tan^{-1}\frac{2}{s} + \tan^{-1}\frac{s}{2} = \frac{\pi}{2}$

3 i $L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = L^{-1}\left\{\frac{A}{s-3} + \frac{B}{s-4}\right\}$

$s-5 = A(s-4) + B(s-3)$

when $s=4$

$-1 = B$

$B = -1$

when $s=3$

$-2 = -A$

$A = 2$

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = L^{-1}\left\{\frac{2}{s-3}\right\} - L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= 2e^{3t} - e^{4t}$$

ii $L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = L^{-1}\left\{\frac{A}{s-2} + \frac{B}{s-4}\right\}$

$2s-6 = A(s-4) + B(s-2)$

when $s=4$

$2 = 2B$

$B = 1$

when $s=2$

$-2 = -2A$

$A = 1$

$$L^{-1}\left\{\frac{2s-4}{(s-2)(s-4)}\right\} = L^{-1}\left\{\frac{1}{s-2}\right\} + L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= e^{2t} + e^{4t}$$

iii $$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = L^{-1}\left\{\frac{A}{s} + \frac{B}{s-4}\right\}$$

$5s - 8 = A(s-4) + Bs$

when $s = 0$

$-8 = -4A$

$A = 2$

when $s = 4$

$12 = 4B$

$B = 3$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = L^{-1}\left\{\frac{2}{s}\right\} + L^{-1}\left\{\frac{3}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$

iv $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{\cancel{(s+1)}(s-4)}{(s-3)(s-1)^{\cancel{2}}}$$

$s^2 - 3s - 4$

$s^2 + s - 4s - 4$

$s(s+1) - 4(s+1)$

$(s+1)(s-4)$

$$L^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = L^{-1}\left\{\frac{A}{s-3}\right\} + L^{-1}\left\{\frac{B}{s-1}\right\}$$

$s - 4 = A(s-1) + B(s-3)$

when $s = 1$

$-3 = -2B$

$B = 2/3$

when $s = 3$

$-1 = 2A$

$A = -1/2$

$$L^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = L^{-1}\left\{\frac{-1}{2(s-3)}\right\} + \frac{1}{3} L^{-1}\left\{\frac{2}{s-1}\right\}$$

$$= \frac{-1}{2}e^{3t} + \frac{2}{3}e^{t} = \frac{2}{3}e^{t} - \frac{1}{2}e^{3t}$$

v $L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} = L^{-1}\left\{\frac{s-5}{(s+2)^2+16}\right\}$

$= L^{-1}\left\{\frac{s}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$

$= L^{-1}\left\{\frac{s+2-2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$

$= L^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$

$= L^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{7}{(s+2)^2+16}\right\}$

$= L^{-1}\left\{\frac{s+2}{(s+2)^2+4^2}\right\} - L^{-1}\left\{\frac{7}{(s+2)^2+4^2} \times \frac{4}{4}\right\}$

$= e^{-2t}\cos 4t - \frac{7}{4} L^{-1}\left\{\frac{4}{(s+2)^2+4^2}\right\}$

$= e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$