NGOCHINDO JIMA FESTUS
18/EN001/008

# CHE 531 ASSIGNMENT IV

1) From Figure 1

$$\bar{y}(s) = \frac{\frac{1}{\tau_p s+1} \cdot \frac{1}{\tau_f s+1} G_c(s)}{1+\frac{1}{\tau_p s+1} \cdot \frac{1}{\tau_f s+1} G_c(s) \frac{1}{\tau_m s+1}} \cdot y_{sp}(s)$$

$$1+\frac{1}{\tau_p s+1} \cdot \frac{1}{\tau_f s+1} G_c(s) \frac{1}{\tau_m s+1} = 0$$

$$1+\frac{1}{s+1} \cdot \frac{1}{\frac{1}{2}s+1} \cdot k_c \frac{1}{\frac{1}{3}s+1} = 0$$

$$1+\frac{1}{s+1} \cdot \frac{2}{s+2} \cdot \frac{k_c}{1} \cdot \frac{3}{s+3} = 0$$

$$1+\frac{6k_c}{(s+1)(s+2)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s+3)+6k_c}{(s+1)(s+2)(s+3)} = 0$$

$$(s+1)(s+2)(s+3)+6k_c = 0$$

Expanding we have

$$s^3+6s^2+11s+6+6k_c = 0$$

Applying Routh's stability approach

Row 1: $a_0$ $a_2$
Row 2: $a_1$ $a_3$
Row 3: $b_1$ $b_2$
Row 4: $c_1$

$$\underset{a_0}{s^3} + \underset{a_1}{6s^2} + \underset{a_2}{11s} + \underset{a_3}{6+6k_c}$$

Row 1 : 1 11

Row 2 : 6 $6+6k_c$

Row 3 : $b_1$ $b_2$

Row 4 : $c_1$

$$b_1 = \frac{6\times 11 - 6+6k_c}{6} = \frac{\cancel{66} - \cancel{6+6k_c}}{6} = 60$$

$$b_1 = \frac{66-6-6k_c}{6} = \frac{60-6k_c}{6} = 10-k_c$$

$$b_1 = 10-k_c$$

$$c_1 = \frac{10-k_c \times 6+6k_c - 6(0)}{10-k_c} = 6+6k_c - 6(0)$$

$$c_1 = 6+6k_c$$

For the system to be stable

$$b_1 = 10-k_c > 0 \qquad c_1 = 6+6k_c > 0$$

$$\cancel{-}k_c > \cancel{-}10 \qquad 6k_c > -6$$

$$k_c < 10 \qquad k_c > -1$$

$$-1 < k_c < 10$$

2.) $$1 + \frac{1}{\tau_p s+1}\,\frac{1}{\tau_p s+1}\,G_{c(s)}\,\frac{1}{\tau_m s+1} = 0$$

where ~~$G_{c(s)}$~~ $k_c = 5$ $\tau_I = 0.25$

For PI controller $G_c = k_c + \frac{k_c}{\tau_I s}$

$$1 + \frac{1}{s+1}\cdot\frac{1}{\frac{1}{2}s+1}\left(5 + \frac{5}{0.25s}\right)\frac{1}{\frac{1}{3}s+1} = 0$$

$$1 + \frac{1}{s+1}\cdot\frac{2}{s+2}\,\cancel{k_c}\left(5 + \frac{20}{s}\right)\frac{3}{s+3} = 0$$

$$1 + \frac{1}{s+1} \cdot \frac{2}{s+2} \left( \frac{5s+20}{s} \right) \frac{3}{s+3} = 0$$

$$1 + \frac{6(5s+20)}{(s+1)(s+2)(s)(s+3)} = 0$$

$$1 + \frac{30s + 120}{(s+1)(s+2)(s)(s+3)} = 0$$

$$\frac{(s+1)(s+2)(s)(s+3) + 30s + 120}{(s+1)(s+2)(s)(s+3)} = 0$$

$$(s+1)(s+2)(s)(s+3) + 30s + 120 = 0$$

$$(s)(s+1)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^2 + s)(s+2)(s+3) + 30s + 120 = 0$$

$$(s^3 + 2s^2 + s^2 + 2s)(s+3) + 30s + 120 = 0$$

$$s^4 + 2s^3 + s^3 + 2s^2 + 3s^3 + 6s^2 + 3s^2 + 6s + 30s + 120$$
$$= 0$$

$$s^4 + 6s^3 + 11s^2 + 36s + 120 = 0$$

Applying Routh's stability approach

| | | | | | |
|---|---|---|---|---|---|
| Row 1 ; | 1 11 | 1 | 11 | 120 |
| Row 2 ; | 6 36 | 6 | 36 | 0 |
| Row 3 ; | 120 b2 | 5 | 120 | 0 |
| Row 4 ; | B1 | -108 | 0 | |
| Row 5 ; | | 120 | | |

$$A_1 = \frac{6 \times 11 - 1 \times 36}{6} = \frac{66 - 36}{6} = 5$$

$$A_2 = \frac{6 \times 120 - 1 \times 0}{6} = \frac{720 - 0}{6} = 120$$

$$A_3 = 0$$

$$B_1 = \frac{5 \times 36 - 6 \times 120}{5} = \frac{180 - 720}{5} = -108$$

Scanned by CamScanner

$B_2 = 0$ , $B_3 = 0$

$$C_1 = \frac{(-108)(120) - (5)(0)}{-108}$$

$$= 120$$

$C_2 = 0$ , $C_3 = 0$

Not all values in the first column is positive therefore the system is Unstable. In order for the system to be stable, all the values in the first column must be positive.