

$$4-s = A(4-s) + B(4-s)$$

$$B = -1$$

$$s = 3$$

$$3-s = A(3+4)$$

$$A = 2$$

$$= \frac{2}{s-3} - \frac{1}{s-4} = 2 \left(\frac{1}{s-3} \right) - \frac{1}{s-4}$$

$$= 2e^{3t} - e^{4t}$$

$$11.) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

$$s = 2$$

$$A = 1$$

$$s = 4$$

$$2 = 2B$$

$$B = 1$$

$$= \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$$

$$111.) \frac{s-8}{s(s-4)} = A \frac{1}{s} + B \frac{1}{s-4}$$

$$y = y_0 + x(y'_0) + \frac{x^2}{2!} (y''_0) + \frac{x^3}{3!} (y'''_0) + \frac{x^4}{4!} (y^{(4)}_0) + \frac{x^5}{5!} (y^{(5)}_0)$$

$$y = y_0 + x(y'_0) + \frac{x^2}{2} \cdot 2(y) + \frac{x^3}{6} \cdot 3(y') + \frac{x^4}{24} \cdot 16(y) + \frac{x^5}{120} \cdot 1824$$

$$y = (y_0)_0 \left(1 + x^2 - \frac{2}{3} x^4 \right) + (y'_0)_0 \left(x + \frac{x^3}{2} + \frac{76}{5} x^5 \right)$$

$$\begin{aligned} 2.) \quad & 3e^{-4t} - 5e^{4t} \\ &= 3 \cdot \frac{1}{s+4} - 5 \cdot \frac{1}{s-4} \\ &= \frac{3}{s+4} - \frac{5}{s-4} \end{aligned}$$

$$\begin{aligned} b.) \quad & \sin 4t + \cos 4t \\ &= \frac{4}{s^2+4} + \frac{5}{s^2+4^2} = \frac{4}{s^2+16} \end{aligned}$$

$$\begin{aligned} c.) \quad & t^3 + 2t^2 - t + 4 \\ & \frac{3!}{s^4} + 2 \left(\frac{2!}{s^3} \right) - \frac{1}{s^2} + \frac{4}{s} \\ &= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} \end{aligned}$$

$$\begin{aligned} d.) \quad & e^{-2t} \cos t \\ &= \frac{(s+2)}{(s+2)^2 + 1} = \frac{s+2}{(s+2)^2 + 1} = \frac{s+2}{(s+2)^2 + 2s} \end{aligned}$$

$$\begin{aligned} e.) \quad & \tan 3t \\ &= \frac{-d}{ds} \left(\frac{3}{s^2+9} \right) = -\frac{d}{ds} \left(\frac{3}{s^2+9} \right) \\ & \frac{d}{ds} \left(3 (s^2+9)^{-1} \right) = 3 \frac{d}{ds} (s^2+9)^{-1} \\ & \frac{d}{ds} = \frac{(s^2+9)_0 \cdot s(2s)}{(s^2+9)^2} = \frac{-9s}{(s^2+9)^2} \end{aligned}$$

$$\text{v.) } \frac{s-5}{s^2+4s+20}$$

$$\frac{s-5}{s^2+4s+20} = \frac{As+B}{s^2+4s+20}$$

$$s-5 = As+B$$

$$A=1, B=-5$$

$$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+4)^2+4^2}$$
$$= e^{st} \cos 4t$$

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15/EN061002

Mechanical

~~Water~~

$$1.) (1-x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2} u^{n-2} v'' + n(n-1)(n-2) u^{n-3} v''' + (1-x^2) y^{n-2xy} + 3y = 0$$

$$\text{for } (1-x^2) y'' \text{, } v = 1-x^2 \quad v' = -2x \quad v'' = -2$$

$$= (1-x^2) y^{n+2} - n y^{n+2-1} (-2x) + \frac{n(n-1)}{2} y^{n+2-2} + 0$$
$$= (1-x^2) y^{n+2} + 2x n y^{n+1} - \frac{n(n-1)}{2} y^n$$

$$\text{for } -2xy$$

$$v = -2x \quad v' = -2 \quad v'' = 0$$

$$u^n = y^{n+1}$$
$$= -2xy^{n+1} + n y^{n+1-1} (-2) + 0$$
$$= -2xy^{n+1} - 2n y^n$$

for 2y

$$v = 2 \quad v' = 0$$

$$y^n = u^n$$
$$= 2y^n$$

$$y^n = (1-x^2) y^{n+2} + 2x n y^{n+1} - \frac{n(n-1)}{2} y^n - 2x y^{n+1} - 2n y^n$$

$$\text{at } x=0$$

$$\frac{1}{2} y^{n+2} + 2y^n - (n^2 + n) y^n - 2n y^n$$

$$8) \frac{e^{-t} - e^{2t}}{t}$$

$$L(e^{-t}) = \frac{1}{s+1}; \quad L(e^{-2t}) = \frac{1}{s+2}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = (-1)^+ \frac{d}{ds} \left[\frac{1}{s+1} - \frac{1}{s+2} \right]$$

$$= \frac{2s+3}{(s^2+3s+2)^2}$$

$$9) e^{4t} \cos 2t$$

$$= \frac{s-4}{(s-4)^2 + 4}$$

$$10) t \sin 2t$$

$$L(t \sin 2t) = -\frac{d}{ds} \left(\frac{2}{s^2+4} \right) = \frac{2 \, ds}{ds} (s^2+4)^{-2}$$

$$\frac{d}{ds} \left(\frac{2}{s^2+4} \right) = \frac{(s^2+4) \cdot 0 - 2(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$11) t^3 + 4t^2 + 5$$

$$= \frac{3!}{s^4} + 4 \left(\frac{2!}{s^3} \right) + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$A = 2$$

$$= \frac{2}{s} + \frac{3}{s-4}$$

$$= 2 + 3e^{4t}$$

$$10) \frac{(s^2 - 3s - 4)}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

$$s = 3$$

$$3^2 - 3(3) - 4 = A(3-1)^2$$

$$\frac{-4}{4} = \frac{4A}{4} \quad A = -1$$

$$s = 1$$

$$1^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$$

$$\frac{-6}{-2} = \frac{-2}{-2} \quad C = ?$$

$$s = 0$$

$$0^2 - 3(0) - 4 = A(0-1)^2 + B(0-3)(0-1) + C(0-3)$$

$$-4 = A + 3B - 3C$$

$$-4 = -1 + 3B - 3(3)$$

$$-4 = -1 + 3B - 9$$

$$-4 = -1 + 3B - 9$$

$$-4 = -10 + 3B$$

$$-4 + 10 = 3B \quad B = 2$$

$$\frac{6}{3} = \frac{3B}{3}$$

$$= \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$= -e^{3t} + 2e^t + 5te^t$$