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15[illegible]
Chemical Engineering

2)

i) $3e^{-4t} - 5e^{4t}$

$L[3e^{-4t}] - L[5e^{4t}]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4+s}{(s^2+16)}$

iii) $t^3 + 2t^2 - t + 4$

$L[t^3] + L[2t^2] - L[t] + L[4]$

$= \frac{3!}{s^{3+1}} + \frac{2 \cdot 2!}{s^{2+1}} - \frac{1}{s} + \frac{4}{s}$

$= \frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s} + 4/s$

iv) $e^{-2t} \cos 5t$

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t} \cos 5t] = \frac{s+2}{(s+2)^2+25}$

v) $t \sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t \sin 3t] = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$

$L[t \sin 3t] = \frac{(-)-6s}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$\lim_{t=0} = \frac{e^{-(0)} - e^{-2(0)}}{0} = \frac{1-1}{0} = \frac{0}{0}$

Using L'Hôpital's rule.

$\lim_{t \to 0} = \left[\frac{-e^{-t} + 2e^{-2t}}{1}\right] = \frac{-1+2}{1} = 1$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{s=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2} \, ds$

vii) $e^{4t} \cos 2t$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t} \cos 2t] = \frac{s-4}{(s-4)^2+4}$

viii) $t \sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t \sin 2t] = -\frac{d}{ds}\left(2/s^2+4\right)$

$= (-) - \frac{4s}{(s^2+4)^2}$

$= 4s/(s^2+4)^2$

x) $t^2 + 4t + 5$

$L[t^2] + L[4t] + L[5]$

$= \frac{2!}{s^{2+1}} + \frac{4 \cdot 2!}{s^{2+1}} + \frac{5}{s}$

$= \frac{2}{s^3} + \frac{8}{s^3} + \frac{5}{s}$

$= \frac{10}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2+4)$

$L(t^2+4) = \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)^3}$

xi) $t^2 \cos t$

$L[\cos t] = \frac{s}{s^2+1^2}$

$= \frac{s^2+1^2(1) - s(2s)}{(s^2+1^2)^2}$

$= \frac{s^2+1^2-2s^2}{(s^2+1)^2}$

$= (-)\frac{3s^2+1}{(s^2+1)^2}$

$= -\left[\frac{(s^2+1) \times -6s + (3s^2+1)(4s)}{(s^2+1)^4}\right]$

$= -\left[\frac{6s^3 - 6s + 12s^3 + 4s}{(s^2+1)^4}\right]$

$= -\left[\frac{18s^3 - 2s}{(s^2+1)^4}\right]$

$= \frac{-18s^3 + 2s}{(s^2+1)^4}$

xii) $\frac{\sinh 2t}{t}$

Using L'Hopital rule

$\frac{2\cosh 2t}{1}$

$\lim_{t \to 0} \frac{2\cosh 0}{1} = 2 \quad = \int_{\sigma=s}^{\infty} f(\sigma)$

$f(s) = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$

$= \int_{s=\sigma}^{\infty} = \frac{2}{\sigma^2-4}$

$= 2\int_{s=\sigma}^{\infty} \frac{1}{\sigma^2-4}\, d\sigma$

$= 2\left(\frac{1}{2(2)}\right) \ln\left[\frac{\sigma-2}{\sigma+2}\right]$

$= \frac{1}{2} \ln\left[\frac{\sigma-2}{\sigma+2}\right]\Big|_{s=s}^{\infty}$

$= \frac{1}{2} \ln\left(\frac{\infty-2}{\infty+2}\right) - \frac{1}{2}\ln\left(\frac{s-2}{s+2}\right)$

$= \frac{1}{2} \ln\left(\frac{\infty}{\infty}\right) - \frac{1}{2}\ln\left(\frac{s-2}{s+2}\right)$

$= \ln 1 - \frac{1}{2}\ln\left(\frac{s-2}{s+2}\right)$

$= \ln\left(\frac{s+2}{s-2}\right)^{1/2}$

v) $\dfrac{s-5}{s^2+4s+20} = \dfrac{s-5}{(s+4)(s+5)}$

$$\frac{s-5}{(s+4)(s+5)} = \frac{A}{s+4} + \frac{B}{s+5}$$

A; $s+4=0$; $s=-4$

$$\frac{-4-5}{-4+5} = \frac{-9}{1} = -9$$

B; $s+5=0$; $s=-5$

$$\frac{-5-5}{-5+4} = \frac{-10}{1} = -10$$

$$\frac{A}{s+4} + \frac{B}{s+4} = \frac{-9}{s+4} + \frac{-10}{s+5}$$

$$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+4)(s+5)}$$

$$= -9e^{-4t} + 10e^{-5t}$$

$$= 10e^{-5t} - 9e^{-4t}$$

i) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$$\underbrace{(1-x^2)\,y''}_{w_1} - \underbrace{2x\,y'}_{w_2} + \underbrace{2y}_{w_3} = 0$$

For w_1

let $u = y''$ $\quad V = 1-x^2$

$u^n = y^{n+2}$ $\quad V' = -2x$

$u^{n-1} = y^{n+1}$ $\quad V'' = -2$

$u^{n-2} = y^n$ $\quad V''' = 0$

$$\cancel{y^{n+2}(1-x^2)}^{\,x=0} + \cancel{n y^{n+1} \times -2x}^{\,x=0} + \frac{n(n-1)}{\cancel{2!}}\, y^n \times -\cancel{2}$$

$= -n(n-1)\,y^n + y^{n+2}$

For w_2

$-2xy'$

$u = y_1$ $\quad V = -2x$

$u^n = y^{n+1}$ $\quad V' = -2$

$u^{n-1} = y^n$ $\quad V'' = 0$

$y^{n+1} \times -2x + n y^n \times -2$

$= -2ny^n$

For w_3

$2y$ $\quad V = 2$

$u^n = y^n$ $\quad V' = 0$

$y^n \times 2 = 2y^n$

$w_1 + w_2 + w_3$

$= -n(n-1)\,y^n - 2ny^n + 2y^n + y^{n+2}$

$= y^{n+2} + 2y^n - n(n-1)\,y^n - 2ny^n$

$= y^{n+2} + y^n\,(2 - n(n-1) - 2n)$

$= y^{n+2} + y^n\,(2 - n^2 + n - 2n)$

$y^{n+2} = -y^n(2-n^2-n) = y^n(-2+n^2+n)$

when $n = 0, 1, 2, 3, 4$

$n=0$ $(y^{(2)})_0 = (-y^0)(2-0^2-0)$

$(y^{(2)})_0 = -(y^0)(2)$

$n=1$ $(y^{(3)})_0 = -y^{(1)}(2+(1)^2-1)$

$(y^{(3)})_0 = -y^{(1)}(2+\cancel{1}-\cancel{1})$

$= -y^{(1)}(2)$

$n=2$ $(y^{(4)})_0 = -y^{(2)}(2+(2)^2-2)$

$= -y^{(2)}(\cancel{2}+4-\cancel{2})$

$= -y^{(2)}(4)$

$n=3$ $(y^{(5)})_0 = -y^{(3)}(2-(3)^2-3)$

$= -y^{(3)}(2+9-3)$

$= -y^{(3)}(8)$

$y^{n+2} = y^n(-2+n^2+n)$

$n=0$ $(y^2) = (y^0)(-2+0+0)$

$(y^2) = (y^0)(-2)$

$n=1$ $(y^3) = (y^1)(-2+1+1)$

$= (y^1)(0)$

$n=2$ $(y^4) = (y^{(2)})(-2+(2)^2+2)$

$(y^4) = (y^{(2)})\cdot(4) \Rightarrow 4 \times -2(y^0) = -8(y^0)$

$n=3$ $(y^5) = (y^{(3)})(-2+(3)^2+2)$

$(y^5) = (y^{(3)})(10) \Rightarrow 10(0)y^1 = 0(y^0)$.

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Using maclarin series

$$y = \left(y^{(0)}\right)_0 + x\left(y^{(1)}\right)_0 + \frac{x^2}{2!}\left(y^{(2)}\right)_0 + \frac{x^3}{3!}\left(y^{(3)}\right) + \frac{x^4}{4!}\left(y^{(4)}\right) + \frac{x^5}{5!}\left(y^{(5)}\right)$$

$$= \left(y^{(0)}\right)_0 + x\left(y^{(1)}\right)_0 + \frac{x^2}{2!}\left(y^{(2)}\right)_{(-2)} + \cancel{\frac{x^3}{3!}\left(y(0)\right)} + \frac{x^4}{4!}\left(-8\left(y^0\right)\right) + \cancel{\frac{x^5}{5!}\left(0y^{\cdot}\right)}$$

$$= \left(y^0\right)_0 + x\left(y^{(1)}\right)_0 + \frac{x^2}{\cancel{2}\times 1}\left(y^{(0)}\right)\cancel{-2} + \frac{x^4}{\cancel{4}\times 3\times\cancel{2}\times 1} - \cancel{8}\left(y^{\circ}\right)$$

$$= \left(y^0\right)_0 + x\left(y^{(1)}\right)_0 - x^2\left(y^0\right) - \frac{x^4}{3}\left(y^{\circ}\right)$$

$$= \left(y^0\right)_0 - x^2\left(y^0\right)_0 - \frac{x^4}{3}\left(y^0\right)_0 + x\left(y^{\prime}\right)_0$$

$$= \left(y^0\right)_0\left(1 - x^2 - \frac{x^4}{3}\right) - \left(y^{(1)}\right)_0 (x)$$