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1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

SOLN

For w_1:		For w_2:		For w_3:
$u^0 = y^2$	$v = 1-x^2$	$u = y'$	$v = -2x$	$u = 2y$
$u^n = y^{n+2}$	$v' = -2x$	$u^n = y^{n+1}$	$v' = -2$	$u^n = 2y^n$
$u^{n-1} = y^{n+1}$	$v^2 = -2$	$u^{n-1} = y^n$		
$u^{n-2} = y^n$				

$y^n = y^{n+2}(1-x^2) + n.y^{n+1}.(-2x) + \frac{n(n-1)y^n.(-2)}{2!} - 2x.y^{n+1} - n(2)y^n + 2y^n = 0$

$(1-x^2)y^{n+2} - 2xny^{(n+1)} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n = 0$

$(1-x^2)y^{n+2} - (2n+2)xy^{n+1} - n(n-1) - 2n + 2y^n = 0$

$(1-x^2)y^{n+2} - 2x(n+1)y^{n+1} - n^2 + n - 2n + 2y^n = 0$

$(1-x^2)y^{n+2} - 2x(n+1)y^{n+1} - (n^2+n-2)y^n = 0$

let $x = 0$

$y^{n+2} = (n^2+n-2)y^n$

When $n=0$ $\quad y^2 = -2(y^0)$

" $n=1$ $\quad y^3 = 0$

" $n=2$ $\quad y^4 = 4(y^2) = (4)(-2)(y^0)$

" $n=3$ $\quad y^5 = 10(y^3)_0 = 10(0) = 0$

" $n=4$ $\quad y^6 = 18(y^4) = 18(4)(-2)(y^0)$

" $n=5$ $\quad y^7 = 28(y^5) = 28(0) = 0$

" $n=6$ $\quad y^8 = 40(y^6) = 40(18)(4)(-2)_0(y^0)$

From macalaurin's series

$y = y^0 + x(y')_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 + \frac{x^5}{5!}(y^5)_0 + \frac{x^6}{6!}(y^6) \ldots$

$y = y^0 + x(y') + \frac{x^2}{2!}(-2)(y^0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(y^0)(-2) + \frac{x^5}{5!}(0)$

$+ \frac{x^6}{6!}(18)(4)(-2)(y^0) + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(40)(18)(4)(-2)(y^0)_0$

$$y = y^0 + x(y')_0 - x^2(y^0) - \frac{x^4}{3}(y^0)_0 - \frac{x^6}{5}(y$$

$$y = y^0 \left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} \right] + x(y')$$

2 i) $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

iii) $t^3 + 2t^2 - t + 4$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t} \cos 5t$

$= \frac{s+2}{(s+2)^2 + 25}$

vi] $t \sin 3t$

$L\{\sin 3t\} = \dfrac{3}{s^2+9}$

$L\{t \sin 3t\} = -\dfrac{d}{ds}\left[\dfrac{3}{s^2+9}\right]$

$u = 3$ $\qquad v = s^2+9$

$\dfrac{du}{dx} = 0$ $\qquad dv/dx = 2s$

$= -\left[\dfrac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2}\right]$

$= -\left[\dfrac{-6s}{(s^2+9)^2}\right]$

$= \dfrac{6s}{(s^2+9)^2}$

vii] $\dfrac{e^{-t} - e^{-2t}}{t}$

$L\{e^{-t} - e^{-2t}\} = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$L\left\{\dfrac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma = s}^{\infty}\left(\dfrac{1}{s+1} - \dfrac{1}{s+2}\right) ds$

$= \int_{\sigma = s}^{\infty}\left(\dfrac{1}{s+1}\right) ds - \int_{\sigma = s}^{\infty}\left(\dfrac{1}{s+2}\right) ds$

$= \int_{s}^{\infty}\left(\dfrac{1}{\sigma+1}\right) ds - \int_{s}^{\infty}\left(\dfrac{1}{\sigma+2}\right) ds$

$= \ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\dfrac{\sigma+1}{\sigma+2}\right]_s^{\infty}$

$= \ln\left[\dfrac{\infty+1}{\infty+2}\right] - \ln\left[\dfrac{s+1}{s+2}\right]$

$= \ln 1 - \ln\left[\dfrac{s+1}{s+2}\right]$

$= 0 - \ln\left[\dfrac{s+1}{s+2}\right]$

$= \ln\left[\dfrac{s+1}{s+2}\right]^{-1}$

$= \ln\left[\dfrac{s+2}{s+1}\right]$

viii] $e^{4t} \cos 2t$

$= \dfrac{s-4}{(s-4)^2 + 4}$

viii] $t \sin 2t$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

$L\{t \sin 2t\} = -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

$u = 2$ $\quad v = s^2+4$

$\frac{du}{dx} = 0$ $\quad \frac{dv}{dx} = 2s$

$= -\left[\frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2}\right]$

$= -\left[\frac{-4s}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

ix] $t^3 + 4t^2 + 5$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x] $e^{3t}[t^2+4]$

$= \frac{2}{(s-3)^3} + \frac{4}{s-3}$

xi] $t^2 \cos t$

$L\{\cos t\} = \frac{s}{s^2+1}$

$L\{t \cos t\} = -\frac{d}{ds}\left[\frac{s}{s^2+1}\right]$

$u = s$ $\quad v = s^2+1$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = 2s$

$= -\left[\frac{(s^2+1)(1) - 2s(s)}{(s^2+1)^2}\right]$

$= -\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right]$

$= -\left[\frac{-s^2+1}{(s^2+1)^2}\right]$

$= \frac{s^2-1}{}$

$u = s^2 - 1$ $\quad\quad$ $v = (s^2+1)^2$

$\frac{du}{dx} = 2s$ $\quad\quad$ $\frac{dv}{dx} = 4s[s^2+1]$

$$= \frac{2s(s^2+1)^2 - 4s(s^2+1)(s^2-1)}{(s^2+1)^4}$$

$$= \frac{\cancel{s^2+1}\left[2s(s^2+1) - 4s(s^2-1)\right]}{\cancel{(s^2+1)^4}}$$

$$= \frac{2s\left[s^2+1 - 2s(s^2-1)\right]}{(s^2+1)^3}$$

xii) $\frac{\sinh 2t}{t}$

$L\{\sinh 2t\} = \frac{2}{s^2-4}$

$$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{2}{s^2-4}$$

$$= \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2-4}$$

$$= \ln\left[\frac{2}{\sigma^2-4}\right]_s^{\infty}$$

$$= \ln\left[\frac{2}{\infty^2-4}\right] - \ln\left[\frac{2}{s^2-4}\right]$$

$$= -\ln\left[\frac{2}{s^2-4}\right]$$

$$= \ln\left[\frac{2}{s^2-4}\right]^{-1}$$

$$= \ln\left[\frac{s^2-4}{2}\right]$$

3 i) $L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{s-3} + \frac{B}{s-4}$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

for A : $s-3=0$, $s=3$

B : $s-4=0$, $s=4$

$3-5 = A(3-4) + B(0)$

$-2 = -A$

$A = 2$

$4-5 = A(0) + B(1)$

$B = -1$

$$\frac{s-5}{(s-3)(s-4)} = L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

III $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$$

For A : $s-2=0$, $s=2$

" B : $s-4=0$, $s=4$

$2(2)-6 = A(2-4) + B(0)$

$-2 = -2A$

$A = 1$

$2(4)-6 = A(0) + B(4-2)$

$2 = 2B$

$B = 1$

$$\frac{2s-6}{(s-2)(s-4)} = L^{-1}\left[\frac{1}{s-2} + \frac{1}{s-4}\right]$$

$$= e^{2t} + e^{4t}$$

III $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$$\frac{5s-8}{s(s-4)} = \frac{A(s-4) + Bs}{s(s-4)}$$

For A : $s=0$

" B : $s-4=0$, $s=4$

$5(0)-8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$5(4)-8 = A(0) + 4B$

$12 = 4B$

$B = 3$

iv $\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{(s-1)} + \dfrac{C}{(s-1)^2}$

$\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A(s-1)^2 + B(s-1)(s-3) + C(s-3)}{(s-3)(s-1)^2}$

$(s-4)(s+1) = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$

$(s-4)(s+1) = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$s^2-3s-4 = As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$1 = A+B$

$-3 = -2A - 4B + C$

$-1 = A + 3B - 3C$

$A = 1 - B$

$-3 = -2(1-B) - 4B + C$

$-3 = -2 + 2B - 4B + C$

$-1 = -2B + C$ —— (i)

$-1 = 1 - B + 3B - 3C$

$-2 = +2B - 3C$ —— (ii)

$-1 = -2B + C$

$+ \; -2 = 2B - 3C$

$-3 = -2C$

$C = 3/2$

$-1 = -2B + 3/2$

$2B = 3/2 + 1$

$2B = \frac{5}{2}$

$B = 5/2 \div 2$

$B = 5/4$

$A = 1 - 5/4$

$A = -1/4$

$= \dfrac{-1}{4(s-3)} + \dfrac{5}{4(s-1)} + \dfrac{3}{2(s-1)^2}$

$= \dfrac{-1}{4} e^{3t} + \dfrac{5}{4} e^{t} + \dfrac{3}{2} te^{t}$

$$\frac{s-5}{s^2+4s+20} = L^{-1}\left\{\frac{s-5}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{s}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{s+2-2}{(s+2)^2-16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{5}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} - L^{-1}\left\{\frac{7}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{s+2}{(s+2)^2+4^2}\right\} - L^{-1}\left\{\frac{7}{(s+2)^2+4^2} \times \frac{4}{4}\right\}$$

$$= e^{-2t}\cos 4t - \frac{7}{4} L^{-1}\left\{\frac{4}{(s+2)^2+4^2}\right\}$$

$$= e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$