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MATRIC NO: 15/ENG 02/021

SUBJECT: ENG381

1)

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Matric No. 15/ENG02/021
Subject: ENG381 [Assignment IV]

1) $(1-x^2)\, d^2y/dx^2 - 2x \frac{dy}{dx} + 2y = 0$

Answer

For $(1-x^2)d^2y/dx^2$; $V = (1-x^2)$; $V' = -2x$; $V^2 = -2$; $V^3 = 0$.
$U = d^2y/dx^2 = y''$; $U^n = y^{(n+2)}$.

Using leibnitz theorem

$$y^{(n)} = U^n V + nU^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} U^{(n-3)} V^3 + \cdots$$

$$= y^{(n+2)}\cdot(1-x^2) + ny^{(n+2-1)}\cdot(-2x) + \frac{n(n-1)}{2!} y^{(n+2-2)}\cdot(-2) + \frac{n(n-1)(n-2)}{3!} y^{(n+2-3)}\cdot 0$$

$$= (1-x^2)y^{(n+2)} - 2x\, ny^{(n+1)} - \frac{2n(n-1)}{2\times1} y^n + 0$$

$$= (1-x^2)y^{(n+2)} - 2x\, ny^{(n+1)} - n(n-1)y^n$$

$\therefore (1-x^2)d^2y/dx^2 = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n$

ii) For $2x\, dy/dx$; $V = 2x$; $V' = 2$; $V^2 = 0$
$U = dy/dx = y'$; $U^n = y^{(n+1)}$

Using leibnitz theorem

$$y^n = U^n V + nU^{(n-1)} V' + \frac{n(n-1)}{2!} U^{(n-2)} V^2 + \frac{n(n-1)(n-2)}{3!} U^{(n-3)} V^3$$

$$= y^{(n+1)}\cdot 2x + ny^{(n+1-1)}\cdot 2 + \frac{n(n-1)}{2!} y^{(n+1-2)}\cdot 0 + 0$$

$$= 2xy^{(n+1)} + 2ny^{(n)}$$

$\therefore 2x\, dy/dx = 2xy^{(n+1)} + 2ny^{(n)}$

iii) For $2y$; $v = 2$; $v' = 0$

$u = y$; $u^n = y^n$

Using leibnitz theorem

$$y^n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v^2 + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v^3 + \dots$$

$$= y^n . 2 + n y^{(n-1)} . 0$$

$$= 2y^n$$

$\therefore$ $2y : 2y^n$.

$$\therefore (1-x^2) d^2y/dx^2 - 2x\, dy/dx + 2y = (1-x^2) y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n - 2xy^{(n+1)} + 2ny^{(n)}$$

Taking like terms.

$$= (1-x^2) y^{(n+2)} + y^{(n+1)} (-2xn - 2x) + y^n (-n(n-1) + 2n)$$

$$= (1-x^2) y^{(n+2)} + y^{(n+1)} (-2xn - 2x) + y^n (-n^2 + n + 2n)$$

when $x = 0$,

$$= (1-(0)^2) y^{(n+2)} + y^{(n+1)} (-2(0)n - 2(0)) + y^n (-n^2 + n + 2n)$$

$$\Rightarrow y^{(n+2)} + y^{(n+1)}(0) + y^n (-n^2 + n + 2n)$$

$$\Rightarrow y^{(n+2)} + y^n (-n^2 + n + 2n)$$

$$\Rightarrow y^{(n+2)} = -y^n (-n^2 + n + 2n)$$

Using Maclaurin's theorem

at $n = 0$,

$$y^{(0+2)} = -y^{(0)} (-(0)^2 + 0 + 2(0))$$

$$y^2 = -y^0 (-0 + 0 + 0)$$

$$y_0^2 = 0$$

at $n = 1$

$$y^{(1+2)} = -y^{(1)} (-(1)^2 + 1 + 2(1))$$

$$y_0^3 = -y' (-1 + 1 + 2)$$

$$y_0^3 = -2y_0$$

at $n = 2$

$$y^{(2+2)} = -y^{(2)} (-(2)^2 + 2 + 2(2))$$

$$y_0^4 = -y_0^2 (-4 + 4 + 2)$$

$$y_0^4 = -2y_0^2$$

at $n=3$

$$y^{(3+2)} = -y^{(3)}\left(-(3)^2 + 3 + 2(3)\right)$$

$$\Rightarrow y^{5}_{0} = -y^{3}_{0}(-9 + 3 + 6)$$

$$\Rightarrow y^{5}_{0} = 0$$

at $n = 4$

$$y^{(4+2)} = -y^{(4)}\left(-(4)^2 + 4 + 2(4)\right)$$

$$y^{6} = -y^{4}(-16 + 4 + 8))$$

$$y^{6} \Rightarrow 4y^{4}_{0}$$

Maclaurin's : $y^{n} = y_0 + xy'_0 + \frac{x^2}{2!} y^{2}_{0} + \frac{x^3}{3!} y^{3}_{0} + \frac{x^4}{4!} y^{4}_{0} + \frac{x^5}{5!} y^{5}_{0}$

2) $L\{3e^{-4t} - 5e^{4t}\}$

answer

$L\{3e^{-4t}\} - L\{5e^{4t}\}$

$= L\{3e^{-4t}\} = \int_0^\infty 3e^{-4t}\, e^{-st}\, dt$

$\Rightarrow 3\int_0^\infty e^{-4t} \cdot e^{-st}\, dt$

$\Rightarrow 3\int_0^\infty e^{(-4-s)t}\, dt$

$\Rightarrow \frac{3}{-(s+4)}\, e^{(-4-s)t}\Big|_0^\infty$

$\Rightarrow \frac{3}{-(s+4)}\left(e^{(-4-s)\infty} - e^{(-4-s)0}\right)$

$= \frac{3}{-(s+4)}(0-1)$

$L\{3e^{-4t}\} \Rightarrow \frac{3}{s+4}$ //

$L\{5e^{4t}\} = \int_0^\infty 5e^{4t}\, e^{-st}\, dt$

$= 5\int_0^\infty e^{4t} \cdot e^{-st}\, dt$

$\Rightarrow 5\int_0^\infty e^{(4-s)t}\, dt$

$\Rightarrow \frac{5}{-(s-4)}\, e^{(4-s)t}\Big|_0^\infty$

$\Rightarrow 5 \quad \left(e^{(4-s)\infty} - e^{(4-s)0}\right)$

$\Rightarrow \dfrac{5}{-(s-4)}(0-1)$

$= 5\left(\dfrac{1}{s-4}\right) \qquad = \dfrac{5}{s-4}$ //

$\therefore L\{3e^{-4t} - 5e^{4t}\} = \dfrac{3}{s+4} - \dfrac{5}{s-4}$ //

b) $L\{\sin 4t + \cos 4t\}$

$= L\{\sin 4t\} = \dfrac{4}{s^2+4^2} = \dfrac{4}{s^2+16}$

$= L\{\cos 4t\} = \dfrac{s}{s^2+4^2} = \dfrac{s}{s^2+16}$

$\therefore L\{\sin 4t + \cos 4t\} = \dfrac{4}{s^2+16} + \dfrac{s}{s^2+16}$ //

c) $L\{t^2 + 2t^2 - t + 4\}$

$= L\{t^2\} = \dfrac{2!}{s^{2+1}} = \dfrac{2!}{s^3}$ //

$= L\{2t^2\} = 2\left[\dfrac{2!}{s^3}\right] = 2\left[\dfrac{2!}{s^3}\right]$

$= L\{t\} = \dfrac{1!}{s^{1+1}} = \dfrac{1!}{s^2}$

$+ L\{4\} = \dfrac{4}{s}$

$\therefore L\{t^2 + 2t^2 - t + 4\} = \dfrac{2!}{s^3} + 2\left[\dfrac{2!}{s^3}\right] - \dfrac{1!}{s^2} + \dfrac{4}{s}$ //

d) $L\{e^{-2t}\cos 5t\}$

$= \int_0^\infty e^{-2t}\cos 5t \, e^{-st}\, dt$

$= \dfrac{5}{\ldots} = \dfrac{5}{\ldots} = \dfrac{5}{(\ldots)^2+25} = \dfrac{5}{s^2+2s+4+25}$

2 e) $L\{t \sin 3t\} = (-1)^1 \frac{d}{ds}\left[\frac{3}{s^2+3^2}\right]$

$= (-1)^1 \frac{d}{ds}\left[\frac{3}{s^2+9}\right]$,,

Using $y = \frac{u}{v} = \frac{Vdu - udv}{V^2}$

$u = 3;\ du/ds = 0$

$V = s^2+9;\ dv/ds = 2s$

$= \frac{(s^2+9)\cdot 0 - 3(2s)}{(s^2+9)^2}$

$= -1\left[\frac{-6s}{(s^2+9)^2}\right]$

$L\{t \sin 3t\} \Rightarrow \frac{6s}{(s^2+9)^2}$,,

f) $L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = L\{e^{-t}\} - L\{e^{-2t}\} \div L\{t\}$

$\Rightarrow L\{e^{-t}\} = \frac{1}{s+1}$

$\Rightarrow L\{e^{-2t}\} = \frac{1}{s+2}$

$\Rightarrow L\{t\} = \frac{1!}{t^{1+1}} = \frac{1!}{t^2}.$

i) $L\{t^2+4t^2+5\}$

$$= L\{t^2\} + L\{4t^2\} + L\{5\}$$

for $L\{t^2\} = \frac{2!}{t^{2+1}} = \frac{2!}{t^3}$ //

for $L\{4t^2\} = 4\left[\frac{2!}{t^3}\right]$

for $L\{5\} = \frac{5}{s}$

$$L\{t^2+4t^2+5\} = \frac{2!}{t^3} + 4\left[\frac{2!}{t^3}\right] + \frac{5}{s}$$ //

ii) $L\{e^{5t}(t^2+4)\} =$

) $L\{t^2\cos t\} = \frac{-d}{d s}\left(\frac{-d}{ds}\left(\frac{s}{s^2+1}\right)\right)$

$$= \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2} = \frac{(s^2+1) - 2s^2}{(s^2+1)^2}$$

$$= \frac{\cancel{(s^2+1)} - 2s^2}{(s^2+1)(s^2+1)} \Rightarrow \frac{-2s^2}{(s^2+1)}$$

$$\Rightarrow \frac{d}{ds}\left(\frac{-2s^2}{(s^2+1)}\right)$$

$$\Rightarrow \frac{(s^2+1)(-4s) - (-2s^2)(2s)}{(s^2+1)^2}$$

$$\Rightarrow \frac{-4s^3 - 4s + 4s^3}{(s^2+1)^2}$$

$$L\{t^2\cos t\} = \frac{-4s}{(s^2+1)^2}$$

) $L\left\{\frac{\sinh 2t}{t}\right\} = L\{\sinh 2t\} = \frac{2}{s^2-4}$

$$= \int_0^\infty \frac{1}{\sigma^2} - 4$$

$$= 2\int_{\sigma=s}^\infty \frac{-1}{-1} \quad \frac{1}{\sigma - 4} \, d\sigma$$

$$= -2\int_{\sigma=s}^\infty \frac{1}{2^2 - \sigma^2} \, d\sigma$$

$$= -2\left(\frac{1}{2}\tan^{-1}\frac{\sigma}{2}\right)$$

3) convert the followings to s domain

i) 1 a) $\frac{s-5}{(s-3)(s-4)}$

$$X(s) = \frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\Rightarrow \frac{s-5\,(s-3)(s-4)}{(s-3)(s-4)} = \frac{A(s-3)(s-4)}{(s-3)} + \frac{B(s-3)(s-4)}{(s-4)}$$

$$\Rightarrow s-5 = A(s-4) + B(s-3)$$

Let $s=4$

$$\Rightarrow 4-5 = A\cancel{(4-4)} + B(4-3)$$

$$\Rightarrow -1 = 0 + 1B$$

$$B = -1$$

Let $s=3$

$$\Rightarrow 3-5 = A(3-4) + B\cancel{(3-3)}$$

$$\Rightarrow -2 = -1A + 0$$

$$\Rightarrow A = 2$$

$$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$X(s) \Rightarrow \frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$x(t) = 2e^{3t} - e^{4t}$$

b) $\frac{2s-6}{(s-2)(s-4)}$

$$= \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$X(s) = \frac{2s-6\,(s-2)(s-4)}{(s-2)(s-4)} = \frac{A(s-2)(s-4)}{(s-2)} + \frac{B(s-2)(s-4)}{(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

Let $s=4$

$8-6 = 0 + 2B$

$2 = 2B$

$B = 1$

Let $s = 2$

$2(2)-6 = A(2-4) + B(2-2)$

$4-6 = A(2-4)$

$\Rightarrow -2 = -2A$

$A = 1$ //

$\therefore$

$$X(s) = \frac{2s-6}{(s-2)(s-4)} = \frac{1}{(s-2)} + \frac{1}{(s-4)}$$

$x(t) = e^{2t} + e^{4t}$ //

c) $\frac{5s-8}{s(s-4)}$

$$= \frac{A}{s} + \frac{B}{(s-4)}$$

$$X(s) = \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$\Rightarrow \frac{5s-8\,s(s-4)}{s(s-4)} = \frac{A\,s(s-4)}{s} + \frac{B\,s(s-4)}{s-4}$$

$5s-8 = A(s-4) + B(s)$

Let $s = 4$

$5(4)-8 = A\overset{\to 0}{(4-4)} + B(4)$

$20-8 = 0 + 4B$

$12 = 4B$

$B = 3$ //

Let $s = 0$

$5(0)-8 = A(0-4) + \overset{\to 0}{B(0)}$

$$\therefore X(s) = \frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$x(t) = 2 + 3e^{4t}$$

iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2}$

$$= \frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \cancel{\frac{C}{(s-1)^2}}$$

$$X(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2}(s-3)(s-1)^2 = \frac{A}{s-3}(s-3)(s-1)^2 + \frac{B}{(s-1)^2}(s-3)(s-1)^2$$

$$\Rightarrow s^2 - 3s - 4 = A(s-1)^2 + B(s-3)$$

Let $s = 3$

$$\Rightarrow \cancel{3^2 - 3(3) = A(3-1)^2 + B(3-3)} \quad 3^2 - 3(3) - 4 = A(3-1)^2 + B\overset{\to 0}{(3-3)}$$

$$\Rightarrow \cancel{9 - 3 - 4 = A(2)^2 + 0} \qquad \Rightarrow 9 - 9 - 4 = A(4)$$

$$\Rightarrow \cancel{6 = A(4)} \qquad \Rightarrow -4 = 4A$$

$$\cancel{A = \frac{2}{4} = \frac{2}{4} = \frac{1}{2}} \qquad A = -1$$

Let $s = 1$

$$\Rightarrow \cancel{1^2 - 3 = A(1-1)^2 + B(1-3)} \quad (1)^2 - 3(1) - 4 = A(1-1)^2$$

$$\Rightarrow 1 - 3 = B(1-3)$$

$$\Rightarrow -2 = B(-2)$$

$$B = 1$$

$$\therefore X(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{(s-3)} + \frac{1}{(s-1)^2}$$

$$X(t) = -e^{3t} + te^{t}$$

v) $\dfrac{s-5}{s^2+4s+20}$ — Factorizing: $s^2+4s+20$

$$= \frac{s-5}{(s+2)^2+16}$$

X

$$= \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$\Rightarrow \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2}$$

$$\Rightarrow \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2} \times \frac{4}{4}$$

$$\Rightarrow \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \frac{4}{(s+2)^2+4^2}$$

$$x(t) = e^{-2t} \cos 4t - \frac{7}{4} e^{-2t} \sin 4t$$