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# MECHANICAL ENGINEERING

$$(1-x^2) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0$$

$$(1-x^2) y^{(2)} - 2x y^{(1)} + 2y^{(0)} = 0$$

$$\text{let } u^0 = y^{(1)}, u^1 = y^{(2)}, u^2 = y^{(3)}$$

$$v^0 = (1-x^2) \quad v^1 = -2x \quad v^2 = -2$$

$$W_1 = y^{(n+2)} (1-x^2) + n y^{(n+1)} (-2x) + \frac{(-2)(n)(n-1)}{2!} y^n$$

$$W_2 = - (y^{(n+1)} (2x) + n y^{(n)} (2)) \quad 2!$$

$$y^{(n+2)} (1-x^2) - 2x n y^{(n+1)} - n(n-1) y^n - 2x y^{(n+1)} - 2n y^{(n)} = 0$$

$$0 = y^{(n+2)} (1-x^2) + y^{(n+1)} (2xn - 2x) + y^{(n)} (-2n^2 + n(n-1))$$

when  $x=0$

$$y^{(n+2)} (1-0) + y^{(n+1)} (0-0) + y^{(n)} (-n^2 + n(n-1)) = 0$$

$$y^{(n+2)} + y^{(n)} (-n^2 + n(n-1)) = 0$$

$$y^{(n+2)} = y^{(n)} (n^2 - n + 2) = 0$$

$$\text{when } n=0 \quad y^{(0+2)} = y^{(0)} (0^2 - 0 + 2) = y^{(0)} (2) = 2y^{(0)}$$

$$n=1 \quad y_0^3 = y_0^1 (1^2 - 1 + 2) = y_0^1 (2) = 2y_0^1$$

$$n=2 \quad y_0^4 = y_0^2 (2^2 - 2 + 2) = y_0^2 (4) = 4y_0^2$$

$$n=3 \quad y_0^5 = y_0^3 (3^2 - 3 + 2) = y_0^3 (8) = 8y_0^3$$

$$n=4 \quad y_0^6 = y_0^4 (4^2 - 4 + 2) = y_0^4 (18) = 18y_0^4$$

$$n=5 \quad y_0^7 = y_0^5 (5^2 - 5 + 2) = y_0^5 (28) = 28y_0^5$$

$$n=6 \quad y_0^8 = y_0^6 (6^2 - 6 + 2) = y_0^6 (40) = 40y_0^6$$

$$y = y_0^0 + x y_0^1 + \frac{x^2}{2!} y_0^2 + \frac{x^3}{3!} y_0^3 + \frac{x^4}{4!} y_0^4 + \frac{x^5}{5!} y_0^5 + \frac{x^6}{6!} y_0^6$$

$$+ \frac{x^7 \cdot y_0^7}{7!} + \frac{x^8 \cdot y_0^8}{8!}$$

$$y = y_0 + x y_1' + \frac{x^2}{2 \times 1} + \frac{x^4}{4 \times 2 \times 3 \times 1} (18x - 2y_0) + \frac{x^6}{6 \times 5 \times 4 \times 3 \times 2 \times 1} (18 \times 4x - 2y_0)$$

$$\frac{x^8}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} (40x + 18 \times 4x - 2y_0)$$

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$y = y_0 \left( 1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} + \dots \right) + x y_1'$$

$$2.) \quad 3e^{-4t} - 5e^{4t} = f(t)$$

$$L(f(t)) = F(s)$$

$$F(s) = L(3e^{-4t}) - L(5e^{4t})$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

$$y = y''_{(0)} + x y'_{(0)} + \frac{x^2}{2!} (-2y''_{(0)}) + \frac{x^4}{4!} (4x - 2y''_{(0)}) + \frac{x^6}{6!} (18x^4 - 2y''_{(0)}) + \dots$$

$$y = y''_{(0)} \left( 1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7} + \dots \right) + x y'_{(0)}$$

$$2.) \quad f(t) = 3e^{-4t} - 5e^{4t}$$

$$F(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

$$ii.) \quad \sin 4t + \cos 4t = f(t)$$

$$F(s) = \frac{4}{s^2+16} + \frac{5}{s^2+16}$$

$$iii.) \quad f(t) = t^3 + 2t^2 - t + 4$$

$$F(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$iv.) \quad e^{-2t} \cos 5t = \frac{5}{(s+2)^2 + 25}$$

$$v.) \quad t \sin 3t = - \frac{d}{ds} \left( \frac{3}{s^2+9} \right) = \frac{(s^2+9)(0) - (3)(2s)}{(s^2+9)^2}$$

$$= \frac{6s}{(s^2+9)^2}$$

$$vi.) f(t) = e^{-t} - e^{-2t}$$

$$F(s) = \int_0^{\infty} \frac{t}{b+t} db - \int_0^{\infty} \frac{t}{b+2} db$$

$$= (\ln(b+t) - \ln(b+2)) \Big|_0^{\infty}$$

$$= (\ln(\frac{b+t}{b+2})) \Big|_0^{\infty}$$

$$= (-\ln(\frac{s+1}{s+2}) + (\ln(\frac{\infty}{\infty})))$$

$$= \ln(s+2) - \ln(s+1)$$

G.S. = P.D. + C.F.

$$y = P \cdot z$$

$$\text{let } b^2 + 4 = u \quad \frac{du}{db} = 2b ; db = \frac{du}{2b}$$

$$\int_0^{\infty} \frac{2}{4} \frac{du}{2b} = \frac{2}{2b} \ln u = \frac{1}{b} \ln u$$

$$vii.) e^{4t} \cos 2t = \frac{s}{(s-4)^2 + 4}$$

$$viii.) f(t) = t \sin 2t$$

$$F(s) = -\frac{d}{ds} \left( \frac{2}{s^2+4} \right) = \frac{(s^2+4)(0) - (2)(2s)}{(s^2+4)^2} = \frac{\ln(\infty^2+4)}{4} - \frac{\ln(s^2+4)}{6}$$

$$= \frac{4s}{(s^2+4)^2}$$

$$ix.) f(t) = t^3 + 4t^2 + 5$$

$$F(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$x.) f(t) = e^{3t}(t^2+4) = t^2 e^{3t} + 4e^{3t}$$

$$F(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

$$xi.) f(t) = t^2 \cos t$$

$$F(s) = (-1)^2 \frac{d^2}{ds^2} \left( \frac{s}{s^2+1} \right) = \frac{(s^2+1)(1) - (s)(2s)}{(s^2+1)^2}$$

$$= \frac{d}{ds} \left( \frac{-s^2+1}{(s^2+1)^2} \right) = \frac{(s^2+1)^2(-2s) - (-s^2+1)(2s)}{(s^2+1)^4}$$

$$xii.) \sinh 2t$$

$$F(s) = \int_0^{\infty} \frac{2}{b^2+4} db \quad \text{let } u = b^2+4 \quad du = \frac{du}{2b}$$

$$= \frac{1}{2b} \int_0^{\infty} \frac{2}{4} db = \left( \frac{\ln u}{b} \right) \Big|_0^{\infty}$$

$$= \left( \frac{\ln(b^2+4)}{b} \right) \Big|_0^{\infty}$$

$$xiii.) \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s=3 \Rightarrow 3-5 = A(3-4) + 0$$

$$-2 = -A; A=2$$

$$\text{let } s=4; 4-5 = B(4-3)$$

$$-1 = B$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4} = 2e^{3t} - e^{4t}$$

$$3ii) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$\text{if } s=2; 2(2)-6 = A(2-4)$$

$$\rightarrow -2 = -2A; A=1$$

$$\text{if } s=4; 2(4)-6 = B(4-2)$$

$$2 = 2B; B=1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4} = e^{2t} + e^{4t}$$

$$3iii) \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$\text{if } s=0; 5(0)-8 = A(0-4)$$

$$-8 = -4A; A=2$$

$$\text{if } s=4; 5(4)-8 = B(4)$$

$$12 = 4B; B=3$$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4} = 2 + 3e^{4t}$$

$$3iv) f(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3) + C(s-3)(s-1)$$

$$s^2-3s-4 = As^2-2As+A + Bs-3B + Cs^2-4Cs+C$$

$$s^2-3s-4 = s^2(A+C) + s(-2A+B-4C) + A-3B+C$$

$$A+C=1$$

$$-2A+B-4C=-3$$

$$A-3B+C=-4$$

$$A = \frac{1}{3}, B = \frac{5}{3}, C = \frac{4}{3}$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{3(s-3)} + \frac{5}{3(s-1)^2} + \frac{4}{3(s-1)}$$

$$= \frac{1}{3} (-e^{3t} + 5te^t + 4e^t)$$

$$v) \frac{s-5}{s^2+4s+20} = \frac{Ax+B}{s^2+4s+20}$$

$$s-5 = Ax + B(s^2+4s+20)$$

$$Ax + B = 0 \quad B = -Ax$$

$$4Ax + 4B = 1 \quad 4Ax + 4(-Ax) = 1$$

$$20Ax + 20B = -5$$