OLOMOWEWE ZAINAB OMODUNNI
15/ENG01/012
CHEMICAL ENGINEERING
ENG 381 ASSIGNMENT IV

1) use the leibnits-Maclaurin method to determine a series solution for

$$(1-x^2)\frac{\delta^2 y}{\delta x^2} - 2x\frac{\delta y}{\delta x} + 2y = 0.$$

$P_1 = (1-x^2)\,\delta^2y/\delta x^2 = (1-x^2)\,y^{(2)}$

$P_2 = -2x\,\delta y/\delta x = -2x\,y^{(1)}$

$P_3 = 2y = 2y$

For P_1 | for P_2 | for P_3

$u = y^2 \quad v = 1-x^2 \qquad u = y^{(1)} \quad v = -2x \qquad u = y \quad v = 2$

$u^n = y^{(n+2)} \quad v' = -2x \qquad u^n = y^{(n+1)} \quad v' = -2 \qquad u^n = y^n \quad v' = 0$

$u^{(n-1)} = y^{(n+1)} \quad v'' = -2 \qquad u^{n-1} = y^{(n)} \quad v'' = 0$

$y^{(n-2)} = y^n \quad v''' = 0$

$y_n = y_{n(P_1)} + y_{n(P_2)} + y_{n(P_3)}$

$$y_n = u^n v + n u^{(n-1)} v' + \frac{n(n-1)u^{(n-2)}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!}$$

$$y_n = y^{(n+2)}(1-x^2) + n y^{(n+1)}\cdot(-2x) + n(n-1)y^n(-2) + y^{(n+1)}\cdot(-2x) + n(y^n)(-2) + y^n(2)$$

$$y_n = y^{(n+2)}(1-x^2) - 2xny^{(n+1)} - 2(n^2-n)y^n - 2xy^{(n+1)} - 2ny^n + 2$$

$$y_n = y^{(n+2)}(1-x^2) - 2xy^{(n+1)}(n+1) - y^n(n^2 - n + 2n - 2)$$

$$y_n = y^{(n+2)}(1-x^2) - 2xy^{(n+1)}(n+1) - \cancel{2y^n(n^2-1)}\,y^n(n^2+n-2)$$

When $x = 0$

$$y_n = y^{(n+2)}(1) - 0 - \cancel{2y^n(n^2-1)}\,y^n(n^2+n-2)$$

$$y_n = y^{(n+2)} - \cancel{2y^n(n^2-1)}\ y^n(n^2+n-2)$$
$$0 = y^{(n+2)} - \cancel{2y^n(n^2-1)}\ y^n(n^2+n-2)$$
$$y^{(n+2)} = \cancel{-2(n^2-1)y^n} + y^n(n^2+n-2)$$ recurrence relation

when $n=0$

$$y^{(2)} = +(y^{(0)})_0(0+0-2)$$
$$y^{(2)} = -2(y^{(0)})_0$$

when $n=1$

$$(y^{(3)})_0 = -(y^{(1)})_0(1+1-2) = 0(y^{(1)})_0 = 0$$

$n=2$ $$(y^{(4)})_0 = -(y^{(2)})_0(4+2-2) = +4(y^{(2)})_0 = +(4)(-2)(y^{(0)})_0$$

$n=3$ $$(y^{(5)})_0 = -(y^{(3)})_0(9+3-2) = +9(y^{(3)})_0 = +9(0) = 0$$

$n=4$ $$(y^{(6)})_0 = -(y^{(4)})_0(16+4-2) = +18(y^{(4)})_0 = +18(+4)(-2)(y^{(0)})_0$$

when $n=5$ $$(y^{(7)})_0 = -(y^{(5)})_0(25+5-2) = +28(y^{(5)})_0 = +28(0) = 0$$

when $n=6$ $$(y^{(8)})_0 = -(y^{(6)})_0(36+6-2) = +40(y^{(6)})_0 = +40(+18)(+4)(-2)(y^{(0)})_0$$

when $n=7$ $$(y^{(9)})_0 = -(y^{(7)})_0(49+7-2) = +54(y^{(7)})_0 = +54(0) = 0$$

$$y_n = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0$$
$$+ (y^{(5)})_0\frac{x^5}{5!} + (y^{(6)})_0\frac{x^6}{6!} + (y^{(7)})_0\, x^7/7! + (y^{(8)})_0\, x^8/7! + (y^{(9)})_0\, x^9/9! + \cdots$$

$$y_n = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(-2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8)(y^{(0)})_0$$
$$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(144)(y^{(0)})_0 + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(-5760) + x^9/9!\,(0)$$

$$y_n = (y^{(0)})_0\left(1 - x^2 - x^4/3 + x^6 - \frac{x^8}{\ }\right) + x(y^{(1)})$$

2(i) transform ~~the~~ following to laplace (s) domain

(i) $3e^{-4t} - 5e^{4t}$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\sin 4t + \cos 4t$

$\frac{a}{s^2+a^2} + \frac{s}{s^2+a^2} = \frac{4}{s^2+16} + \frac{s}{s^2+4}$

iii) $t^3 + 2t^2 - t + 4$

$L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$

$\frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $e^{-2t} \cos 5t$

$L\{\cos 5t\} = \frac{s}{s^2+25} = \frac{(s+2)}{(s+2)^2+25}$

v) $t \sin 3t$

$L\{\sin 3t\} = \frac{3}{s^2+9}$

$-d/ds(L\{\sin 3t\}) = -d/ds\left(\frac{3}{s^2+9}\right)$

let $u = 3$ $\quad v = s^2+9$

$du = 0$ $\quad dv = 2s$

$\frac{v\,du - u\,dv}{v^2} = \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$

vi) $\frac{e^{-t} - e^{-2t}}{t}$

$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \frac{1-1}{0} = \frac{0}{0}$

~~lim t→0~~

using l'hopitals rule.

$\left[\frac{-e^{-t}+2e^{-2t}}{1}\right] = \frac{-1+2}{1} = \frac{1}{1}$

$L\left\{e^{-t} \cancel{+2}e^{-2t}\right\} = \left[\frac{-1}{s+1} + \frac{1}{s+2}\right]$

$\int_s^\infty \frac{1}{s+1} - \frac{1}{s+2} \, ds$

$\int_s^\infty \frac{1}{s+1} - \frac{1}{s+2} \, ds$

$\int \frac{1}{s+1} ds - \int \frac{1}{s+2} ds$

$\left[\ln(s+1) - \ln(s+2)\right]_s^\infty$

$\left[\frac{\ln(s+1)}{\ln(s+2)}\right]_s^\infty$

$\ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\left(\frac{s+1}{s+2}\right)$

$= \ln(1) - \ln\left(\frac{s+1}{s+2}\right)$

$= -\ln\left(\frac{s+1}{s+2}\right)$

$= \ln\left(\frac{s+2}{s+1}\right)$

vii) $e^{4t}\cos 2t$

$L\{\cos 2t\} = \frac{s}{s^2+4}$

where

$s = (s-4)$

$= \frac{s-4}{(s-4)^2+4}$

viii) $t \sin 2t$

$L\{\sin 2t\} = \frac{2}{s^2+4}$

~~$-\frac{d}{ds}$ where $L\left\{\frac{2}{s^2+4}\right\}$~~

$-\frac{d}{ds} L\{\sin 2t\}$

$= -\frac{d}{ds}\frac{2}{(s^2+4)}$

$u = 2 \qquad v = s^2+4$

$du = 0 \qquad dv = 2s$

$\frac{vdu - udv}{v^2} = \frac{(s^2+4)(0) - 4s}{(s^2+4)^2}$

$= \frac{-4s}{(s^2+4)^2}$

x) t^3+4t^2+5

$L\{t^3\}+L\{4t^2\}+L\{5\}$

$\frac{3!}{s^4}+\frac{4\times 2!}{s^3}+\frac{5}{s}$

$\frac{3!}{s^4}+\frac{8}{s^3}+\frac{5}{s}$

x) $e^{3t}(t^2+4)$

$t^2e^{3t}+4e^{3t}$

$L\{t^2e^{3t}\}+L\{e^{3t}4\}$

$L\{e^{3t}\}=\frac{1}{(s-3)}$

$-d/ds\left(\frac{1}{s-3}\right)$

let $u=1$ $v=s-3$

~~$du=0$ $dv=s$~~

$\frac{vdu-udv}{v^2}$

$\frac{s-3(0)-1(s)}{(s-3)^2}$

$=\frac{-s}{(s-3)^2}$

$L\{4e^{3t}\}$

$\frac{4}{s-3}$

$=\frac{-s}{(s-3)^2}+\frac{4}{s-3}$

xi) $t^2\cos t$

$L\{\cos t\}=\frac{s}{s^2+1}$

$-d/ds\{s/s^2+1\}$

let $u=s$ $v=s^2+1$

$du=1$ $dv=2s$

$\frac{vdu-udv}{v^2}$

$\frac{s^2+1-2s^2}{(s^2+1)^2}$

$=\frac{-s^2+1}{s^4+2s^2+1}$

$-d/ds\left[\frac{+s^2-1}{s^4+2s^2+1}\right]$

let $u=+s^2-1$

$v=s^4+2s^2+1$

$du=+2s$

$dv=4s^3+4s$

$\frac{(s^4+2s^2+1)2s-(s^2-1)(4s^3+4s)}{(s^4+2s^2+1)^2}$

$=\frac{2s^5+4s^4+2s-4s^5-4s^3-4s^3-4s}{(s^4+2s^2+1)^2}$

$=\frac{-2s^5+4s^4-8s^3-2s}{(s^4+2s^2+1)^2}$

xii) $\frac{\sinh 2t}{t} = L\left\{\frac{\sinh 2t}{t}\right\} = L\left\{\frac{\sinh (0)}{t}\right\} = \frac{0}{0}$

lim $t \to 0$

using l'hopitals rule

$L\left\{\frac{2\cosh 2t}{1}\right\} = \frac{2\cosh(0)}{1} = \frac{2}{1} = 2$

$L\left\{\frac{\sinh 2t}{t}\right\} = \frac{2}{s^2-4} = \int_s^\infty \frac{2}{s^2-4} = 2\int_s^\infty \frac{2}{s^2-4}$

$= 2\left[\frac{1}{2}\tan^{-1}\frac{s}{2}\right]_s^\infty = \left[\tan^{-1}\frac{s}{2}\right]_s^\infty$

$= \tan^{-1} 2/s$

3. convert the following to time (t) domains

(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{(s-3)(s-4)} = A(s-4) + B(s-3)$

at $s=4$

$4-5 = A(4-4) + B(4-3)$ $\quad -1 = B \quad B = -1$

at $s=3$

$3-5 = A(3-4) + B(0)$ $\quad -2 = -A \quad A = 2$

$x(t) = L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

xii) $\frac{\sinh 2t}{t} = L\left\{\frac{\sinh 2t}{t}\right\} = L\left\{\frac{\sinh(0)}{t}\right\} = \frac{0}{0}$

$\lim t \to 0$

using l'hopital's rule

$L\left\{\frac{2\cosh 2t}{1}\right\} = \frac{2\cosh(0)}{1} = \frac{2}{1} = 2$

$L\left\{\frac{\sinh 2t}{t}\right\} = \frac{2}{s^2-4} = \int_s^\infty \frac{2}{s^2-4} = 2\int_s^\infty \frac{2}{s^2-4}$

$= 2\left[\frac{1}{2}\tan^{-1}\frac{s}{2}\right]_s^\infty = \left[\tan^{-1}\frac{s}{2}\right]_s^\infty$

$= \tan^{-1} 2/s$

B. convert the following to time (t) domains

(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$\frac{s-5}{(s-3)(s-4)} = A(s-4) + B(s-3)$

at $s=4$

$4-5 = A(4-4) + B(4-3)$ $-1 = B$ $B = -1$

at $s=3$

$3-5 = A(3-4) + B(0)$ $-2 = -A$ $A = 2$

$x(t) = L^{-1}\left[\frac{2}{s-3} - \frac{1}{s-4}\right]$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

at $s=4$

$2(4)-6 = A(0) + B(2)$

$2 = B2 \quad B = 1$

at $s = 2$

$2(2)-6 = A(2-4) + B(0)$

$-2 = -2A \quad A = 1$

$L\left\{\frac{1}{s-2} + \frac{B}{s-4}\right\} = e^{2t} + e^{4t} = x(t)$

iii) $L^{-1}\left[\frac{5s-8}{s(s-4)}\right]$

$x(t) = L^{-1}\left\{\frac{A}{s} + \frac{B}{s-4}\right\} = \frac{5s-8}{s(s-4)}$

$A(s-4) + B(s) = 5s-8$

at $s=4$

$A(4-4) + B(4) = 5(4)-8$

$4B = 12$

$B = 3$

at $s=0$

$A(0-4) + 0 = 5(0)-8$

$-4A = -8$

$A = 2$

$x(t) = L^{-1}\left\{\frac{2}{s} + \frac{3}{s-4}\right\}$

$= 2 + 3e^{4t}$

Scanned by CamScanner

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = L^{-1}\left[\frac{A}{s-3} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}\right]$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

let $s=1$

$$(1)^2 - 3(1) - 4 = A(0) + B(1-3)(0) + C(-2)$$

$$-6 = 2C$$

$$C = 3$$

at $s=3$

$$3^2 - 3(3) - 4 = A(3-1)^2 + B(0)(3-1) + C(0)$$

$$9-9-4 = 4A$$

$$-4 = 4A$$

$$A = -1$$

at $s=0$

$$0-0-4 = A(-1)^2 + B(-3)(-1) + C(-3)$$

$$-4 = A + 3B - 3C$$

$$-4 = -1 + 3B - 3(3)$$

$$-4 = -1 - 9 + 3B$$

$$-4 = -10 + 3B$$

$$-4 + 10 = 3B$$

$$6 = 3B$$

$$B = 2$$

$$L^{-1}\left[\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right]$$

$$= -e^{3t} + 2e^{t} + 3te^{t}$$

v) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{(s+2)^2+16}$$

Scanned by CamScanner

$\frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

$= \frac{s}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$

$= \frac{\cancel{s+2}+2\ s+2-2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$

$\frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$

$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2} \times \frac{4}{4}$

$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \times \frac{4}{((s+2)^2+4^2)}$

$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\left[\frac{4}{(s+2)^2+4^2}\right]$

$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$