OMACHI VICTOR
15/ENG07/037
PETROLEUM
ENG381
ASSIGNMENT IV

1.) $(1-x)^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

Sub module 1:-

$u = y''$ $u^n = y^{(n+2)}$

$v = 1-x^2$, $v' = -2x$, $v'' = -2$

$y^n = y^{(n+2)}\ 1-x^2 + ny^{(n+1)} \cdot -2x + \frac{n(n-1)}{} y^n \cdot -2$

$y^n = (1-x^2)\ y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n$

Sub module 2

$u = y'$ $u^n = y^{(n+1)}$

$v = -2x$ $v' = -2$

$y^{(n)} = -2xy^{(n+1)} + ny^{(n)} - 2$

$= -2xy^{(n+1)} - 2ny^{(n)}$

Sub module 3:

$u = y$ $u^n = y^n$

$v = 2$

$y^{(n)} = 2 \cdot y^{(n)} = 2y^{(n)}$

Combination

$(1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)}$

$(1-x)^2 y^{(n+2)} - (2nx - 2x)y^{(n+1)} - (n^2 + n - 2)y^{(n)} = 0$

at $x = 0$

$y^{(n+2)} = 0 - (n^2 + n - 2)y^{(n)} = 0$

$y^{n+2} = y^n(n^2 + n - 2)$

When $n = 0$

$(y^{(2)})_0 = (y^0) \cdot (-2) = -2(y^{(0)})_0$

When $n = 1$

$(y^{(3)}) = (y^{(1)}) \cdot (0) = 0$

When $n = 2$ $(y^{(4)}) = (y^{(2)}) \cdot (4) = 4x - 2(y^{(0)})_0 = -8(y^{(0)})$

When $n=3$

$(y^{(5)})_0 = (y^{(3)})_0 (10) = 10(y^{(3)})_0 = 10\times 0 = 0$

When $n=4$

$(y^{(6)}) = (y^{(4)}) = (18) = 18(y^{(4)}) = 18x - 8(y^{(4)})_0 = -144(y^{(0)})$

When $n=5$

$(y^{(7)}) = (y^{(5)})(28) = 28\times 0 = 0$

When $n=6$

$(y^{(8)}) = (y^{(8)})_0 (40) = 40x - 144(y^{(0)})_0 = \frac{-5}{80}(y^{(0)})_0$

$y = (y)_0 + x(y)_0 + \frac{x^2}{2!}(y^{(0)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \cdots$

$y = (y)_0 + x(y^{(0)}) - x^2(y^{(0)})_0 + 0 + -x^4(y^{(0)})_0 + 0 - \frac{x^6}{3}(y^{(0)})_0 + 0 - \frac{x^8}{7}(y^{(0)})_0 +$

$y = (y)_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5} - \frac{x^8}{7}\right] + (y')_0(x)$

2.) i.) $L[3e^{-4t} - 5e^{4t}]$

$\frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)}$

$= \frac{3s - 12 - 5s - 20}{s^2 - 16} = \frac{-2s - 32}{s^2 - 16}$

ii.) $L[\sin 4t + \cos 4t]$

$\frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$

iii.) $L[t^3 + 2t^2 - t + 4]$

$= \frac{3!}{s^4} + \frac{2\cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6 + 4s - s^2 + 4s^3}{s^4}$

iv.) $L[e^{-2t}\cos 5t]$

$\cos 5t = \frac{s}{s^2+5^2}$

Let $s = s+2$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2 + 5^2} = \frac{s+2}{s^2 + 4s + 25}$

v.) $L[t\sin 3t]$

$$L[\sin 3t] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$F'(s) = \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$$

Let $u = 3$ $\quad du/ds = 0$

$v = s^2+9$ $\quad \frac{dv}{ds} = 2s$

using quotient rule

$$\frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$$

$$= \frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{0 - 6s}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$$

vi). $\frac{e^{-1t} - e^{-2t}}{t}$

differentiating seperately

$$\frac{-e^{-t} + 2e^{-2t}}{1} = e^{-2t}$$

at $t = 0$

$$= e^{-2(0)} = 1$$