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T: Petroleum Eng
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$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2$

$u = y^2 \quad u^n = y^{(n+2)}$

$v = 1-x^2 \quad v' = -2x \quad v^2 = -2 \quad v^3 = 0$

$w^n = u^n v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}v^{(n-2)}v^2 + \frac{n(n-1)(n-2)}{3!}y^{(n-3)}v^3$

$= y^{(n+2)}(1-x^2) - 2xny^{(n+1)} - n(n-1)y^n$

$= (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2-n)y^n$

$W_2 = -2x\frac{dy}{dx} = -2xy'$

$u = y' \quad u^n = y^{n+1}$

$v = -2x \quad v' = -2 \quad v^2 = 0$

$= \cancel{y^{(n+1)}} \cdot -2x\, y^{(n+1)} \cdot -2x + ny^n - 2 + 0$

$= -2xy^{(n+1)} - 2ny^n$

$W_3 = 2y$

$u = y \quad u^n = y^n$

$v = 2 \quad v' = 0$

$= 2y^n$

$= (1+x^2)y^{(n+2)} - 2xny^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n + 2y^n$

$= (1-x^2)y^{(n+2)} - 2xy^{(n+1)} - y^n(n^2-n+2n-2)$

$= (1-x^2)y^{(n+2)} - (n+1)2xy^{(n+1)} - (n^2+n-2)y^n$

$= (1-x^2)y^{(n+2)}$

at $x = 0$

$= [y^{n+2}]_0 = (n^2+n-2)y^n$

iii) $L[t^3 + 2t^2 - t + 4]$.

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^4} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

iv) $L[e^{-2t}\cos 5t]$.

$L[\cos 5t] = \frac{s}{s^2+25}$

$L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2+25}$

$= \frac{s+2}{(s+2)^2+25}$

v) $t\sin 3t$

$L[\sin 3t] = \frac{3}{s^2+9}$

$L[t\sin 3t] = -1\frac{d}{ds}\left[\frac{3}{s^2+9}\right]^u_v$

$u = 3 \quad du = 0$

$v = s^2+9 \quad dv = 2s$

$-1\left[\frac{0-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

vi) $e^{-t} - e^{-2t}$

$L[e^{-t} - e^{-2t}]$

$= \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{s+1} - \frac{1}{s+2}$

$= \int_{\sigma}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$

$\ln(\sigma+1) - \ln(\sigma+2)$

$= \ln\left[\frac{\sigma+1}{\sigma+2}\right]$

$\ln\left[\frac{s+1}{s+2}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right]$

$\ln\left[\frac{s+1}{s+2}\right]^{-1} = \ln\left[\frac{s+2}{s+1}\right]$

vii) $L[e^{4t}\cos 2t]$

$L[\cos 2t] = \frac{s}{s^2+4}$

$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

viii) $t\sin 2t$

$L[\sin 2t] = \frac{2}{s^2+4}$

$L[t\sin 2t] = -1\frac{ds}{ds}\left(\frac{2}{s^2+4}\right)\frac{u}{v}$

$u = 2 \quad du = 0$

$v = s^2+4 \quad dv = 2s$

$-1\left[\frac{0-4s}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$

ix) $t^3 + 4t^2 + 5$

$L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + \left[\frac{5}{s}\right]$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$