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15/ENG04/030
ELECT/Elect

$(1-x^2)\, d^2y/dx - 2x\, dy/dx + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

$y^{(n)} = U^n V + nv^{(n-1)} V^{(1)} + \frac{n(n-1)}{2!} U(u-2) V^{(2)} + \ldots$

$[y^{(2+n)} \cdot (1-x^2)] + ny^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^{(n)} \cdot (-V)) + [y^{(1+n)} \cdot$
$2x + ny^{(n)} \cdot 2] + [2y^{(n)}] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)}$
$+ 2y^{(n)} = 0$

let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0$

$y^{(2+n)} + y^{(n)}[-n^2+n-2n+2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2-n+2] = 0$

$\boxed{(y^{(2+n)})_0 = -(y^{(n)})_0 \cdot [-n^2-n+2] =}$ reamerence relat

$n=0 \therefore (y^{(1)})_0 = -(y^{(0)})_0 \cdot (2) = -2(y^{(0)})_0$

$n=1 : (y^{(3)})_0 = -(y^{(1)})_0 \cdot [0] = 0$

$n=2 : (y^{(4)})_0 = -(y^{(2)})_0 (-4] = 4(y^{(2)})_0 = (4)(-2)(y^{(0)})_0$

$n=3 \therefore (y^{(5)})_0 = [-y^{(3)}]_0 \cdot [-10] = 10(y^{(3)})_0 = (10)(0) = 0$

$n=4 \therefore (y^{(6)}]_0 = -(y^{(4)}]_0 (-18) = 18[y^{(4)}]_0 = (18)(4)(-2)(y^{(0)})_0$

$n=5 : [y^{(7)}]_0 = -(y^{(5)})_0 [-28) = (28)(0) = 0$

$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \ldots$

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$$y = (y^0)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(-2)(y^{(0)})_0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)(y^{(0)})_0 + \dots$$

$$\dots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)(4)(-2)(y^{(0)})_0 + \frac{x^7}{7!}(0)$$

$$y = (y^0)_0 + x(y^{(1)})_0 - x^2(y^{(2)})_0 - \frac{x^4}{3 \times 1}(y^{(3)})_0 - \frac{x^5}{5}(y^{(3)})_0$$

$$y = (y^0)\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right) + (y^{(1)})_0 [x_0..]$$

(2). (i)

$$3e^{-4t} - 5e^{4t} = f(t)$$

$$L[f(t)] = \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s+4)} = \frac{3s - 12 - 5s - 20}{(s+4)(s-4)}$$

$$= \frac{-2s - 32}{(s+4)(s+4)}$$

(ii) $\sin 4t + \cos 4t = f(x)$

$$L[f(t)] = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{(s^2+4^2)}$$

(iii) $t^3 + 2t^2 - 6 + 4$

$$\frac{n!}{s^{n+1}} = \frac{3!}{s^4} + \frac{2 \cdot 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$= \frac{1}{s^4}\left[6 + 4s - s^2 + 4s^3\right]$$

(v) $t\sin 3t = f(t)$

$L[\sin 3t] = \dfrac{3}{s^2+3^2} = 3/s^2+9$

$L[t\sin 3t] = -\delta/\delta s\, f(x)$

$u=3 \quad v=s^2+9$

$du=0 \quad dv=2s$

$\dfrac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = \left[\dfrac{-6s}{(s^2+9)^2}\right] = \dfrac{6s}{(s^2+9)^2}$

(v) $e^{-2t}\cos 5t = f(t)$

$L[f(t)] = L[\cos 5t] = 5/s^2+5^2$

$L[f(t)] = \dfrac{s+2}{(s+2)^2+5^2}$ //

(vi) $\dfrac{e^{-t}-e^{-2t}}{t}$

$\lim_{t\to 0}\left[\dfrac{-e^{-t}+2e^{-2t}}{1}\right] = 2/1 = 2$

$L[e^{-t}-e^{2t}] = 1/s+1 - 1/s+2$

$L\left[\dfrac{e^{-t}-e^{-2t}}{t}\right] = \int_{f(2)}^{\infty} \left(\left(\dfrac{1}{r+1}\right) - \left(\dfrac{1}{r+2}\right)\right) dr$

$$= \int_{r=s}^{\infty} \frac{1}{r+1}\, \delta r - \int_{r=s}^{\infty} \frac{1}{6+2}\, \delta r$$

$$= \left[\ln(r+1) - 2\ln(r+2)\right]_s^{\infty}$$

$$= \left[\ln \frac{r+1}{(r+2)^2}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{\infty+2} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)^2}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

(vii) $e^{4t}\cos 2t$

$$t[\cos 2t] = \frac{-s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$$

(viii) $t\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$L[t\sin 2t] = -\frac{d}{ds}[f(s)]$$

$$v=2 \quad \delta v=0, \quad v=s^2+4 \quad \delta v = 2s$$

$$\frac{(s^2+4)\cdot 0 - 2(2s)}{(s^2+4)^2} = \left(\frac{-4s}{(s^2+4)^2}\right) \cdot - \quad -\frac{4s}{(s^2+4)^2}$$

(x) $t^3 + 4t^2 + 5 = f(t)$

$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + 5/s = 6/s^4 + 8/s^3 + 5/s$

$= 1/s^4 [6 + 8s + 5s^3]$

x) $e^{3t}(t^2+4)$

$L[t^2+4] = \frac{2!}{s^3} + 4/s = 2/s^3 + 4/s$

$L[e^{3t}] = 2/(s-3)^3 + 4/(s-3) = \frac{4s^2 - 24s + 38}{(s-3)^2}$

(f1) $t^2 \cos t = f(t)$

$L(\cos t) = s/(s^2+1) \quad \therefore L[t^2 \cos t] = -\delta^2/\delta s^2 [s/(s^2+1)]$

$\frac{d}{ds}\left[\frac{s}{s^2+1}\right] = \frac{[s^2+1] \cdot 1 - s(2s)}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$

$\frac{d}{\delta s}\left[\frac{1-s^2}{(s^2+1)^2}\right] \quad \therefore u = 1-s^2 \quad du = -2s$

$v = (s^2+1)^2 \quad \delta v = 4s(s^2+1)$

$v = s^2+1 \quad \delta v = 2s$

$w = v^2 \quad \delta w = 2v$

$\frac{\delta v}{\delta s} \times \frac{\delta w}{\delta v} = 2s \times 2v$

$= 4sv = 4s(s^2+1)$

$$\frac{(s^2+1)^2 \cdot -2s - (1-s^2)\, 4s(s^2+1)}{(s^2+1)^2} = \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)}$$

$$= \frac{-2s[s^2+1-2+2s^2]}{(s^2+1)^3}$$

$$= \frac{-2s[3s^2-1]}{(s^2+1)^3} \quad \therefore L[t^2\cos t] = -\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\therefore \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] = \frac{2s[3s^2-1]}{(s^2+1)^3}$$

(xii) $\frac{\sinh 2t}{t} = f(x)$

$$\lim_{t\to 0}\left[\frac{\sinh 2t}{t}\right] = \frac{2\cosh 2t}{1} = \frac{2}{1} = 2$$

$$L\left[\frac{\sinh 2t}{t}\right] = L[\sinh 2t] = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{r=s}^{\infty} \frac{2}{r^2-4}\, dr = 2\int_{r=s}^{\infty} \frac{1}{r^2-4}\, dr$$

$$\frac{s-5}{(s-3)(s-4)} = f(s) = \frac{A}{s-3} + \frac{B}{s-4}$$

$$A: (s-3)f(s) = \frac{s-5}{(s-3)(s-4)}\Big|_{s=3} \quad \frac{(3-5)}{(3-4)} = 2$$

$$3.\quad s-4\, f(s)) = \left.\frac{s-5}{(s-3)(s-4)}\right|_{s=4} = \frac{4-5}{4-3} = -1$$

$$f(s) = \frac{2}{s-3} - \frac{1}{s-4}$$

$$f(t) = 2e^{3t} - e^{4t}$$

$$\text{i)}\quad \frac{2s-6}{(s-2)(s-4)} = f(s) = \frac{A}{s-2} + \frac{B}{s-4}$$

$$\text{A)}\quad \left.\frac{2s-6}{s-4}\right|_{s=2} = \frac{2(2)-6}{2-4} = 1$$

$$\text{(B)}\quad \left.\frac{2s-6}{s-2}\right|_{s=4} = \frac{2(4)-6}{4-2} = 1$$

$$f(s) = \frac{1}{s-2} + \frac{1}{s-4}$$

$$f(t) = e^{2t} + e^{4t}$$

$$\text{iii)}\quad \frac{5s-8}{s(s-4)} = f(s) = \frac{A}{s} + \frac{B}{s-4}$$

$$\text{A.}\quad \left.\frac{5s-8}{s-4}\right|_{s=0} = \frac{5(0)-8}{0-4} = \frac{8}{4} = 2$$

$$\text{B}\quad \left.\frac{5s-8}{s}\right|_{s=4} = \frac{5(4)-8}{4} = 3$$

$$f(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$f(t) = 2 + 3e^{4t}$$

(iv). $\frac{s^2-3s-4}{(s-3)(s-1)^2} = f(s) = A/_{s-3} + B/_{(s-1)} + C/_{(s-1)}$

$A: \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$

$B = \left.\frac{s^2-3s-4}{s-3}\right|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$

$C = d/_{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$

at $s=1$

$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 0$

$F(s) = -1/_{s-3} + 3/_{(s-1)^2} + 2/_{s-1}$

$f(t) = -e^{3t} + 3te^{t} + 2e^{t}$

v) $\frac{s-5}{s^2+4s+20} = f(s)$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \sim 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$