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15/ENG01/008
CHEMICAL ENGINEERING

1. $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$$\underbrace{(1-x^2)y^2}_{W_1} - \underbrace{2xy'}_{W_2} + \underbrace{2y}_{W_3} = 0$$

$$y^n = u^{(n)}v + nu^{(n-1)}v' + \frac{n(n-1)}{2!}u^{(n-2)}v^2 + \dots$$

For W_1		For W_2		For W_3	
$u = y^2$	$v = 1-x^2$	$u = y'$	$v = -2x$	$u = 2y$	$v = 1$
$u^n = y^{n+2}$	$v' = -2x$	$u^n = y^{n+1}$	$v' = -2$	$u^n = 2y^n$	$v' = 0$
$u^{n-1} = y^{n+1}$	$v'' = -2$	$u^{n-1} = y^n$	$v'' = 0$		
$u^{n-2} = y^{n}$	$v''' = 0$				

$$y^{n+2}\cdot(1-x^2) + ny^{(n+1)}\cdot(-2x) + \frac{n(n-1)y^n}{2!}\cdot(-2) + y^{(n+1)}\cdot(-2x)$$

$$+ny^n\cdot(-2) + [2y^n] = 0$$

$$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - n(n-1)y^n - 2xy^{(n+1)} - 2ny^{(n)} + 2y^n = 0$$

Let $x = 0$

$$y^{(n+2)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$$

$$y^{(n+2)} + y^n[-n(n-1) - 2n + 2] = 0$$

$$y^{n+2} + y^n[-n^2 + n - 2n + 2] = 0$$

$$y^{n+2} + y^n[-n^2 - n + 2] = 0$$

$$y^{n+2} = -y^{(n)}[-n^2 - n + 2]$$

$n = 0$: $y^{(2)} = -y^0 \cdot 2 = -2y^{(0)}$

$n = 1$: $y^{(3)} = -y^{(1)}[0]$

$n = 2$: $y^{(4)} = -y^{(2)}[-4] = 4y^{(2)} = 4[-2][y^{(0)}] = -8y^{(0)}$

$n = 3$: $y^{(5)} = -y^{(3)}[-10] = 10y^{(3)} = 10\cdot 0 = 0$

$n = 4$: $y^{(6)} = -y^{(4)}[-18] = 18y^{(4)} = 18[-8]y^{(0)}$

$n = 5$: $y^{(7)} = -y^{(5)}\cdot[-28] = 28[y^{(5)}] = 28\cdot 0 = 0$

$$y = y^0 + xy' + \frac{x^2}{2!}y^2 + \frac{x^3}{3!}y^3 + \dots$$

$$y = y^\circ + xy' + \frac{x^2}{2!}(-2)y^\circ + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^\circ + \frac{x^5}{5!}(0)$$
$$+ \frac{x^6}{6!}(18)(-8)y^\circ + \frac{x^7}{7!}(0)$$

$$y = y^\circ + xy' - x^2 y^\circ - \frac{x^4}{3\times 1}y^\circ - \frac{x^6}{5}y^\circ$$

$$y = y^\circ\left[1 - x^2 - x^4/3 - x^6/5\right] + y'_0\,[x]$$

2.i $3e^{-4t} - 5e^{4t}$

$$L\{f(t) \pm G(t)\} = L\{f(t)\} \pm L\{\cancel{f(t)}\} \pm L\{G(t)\}$$

$$L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\}$$
$$\Rightarrow 3L\{e^{-4t}\} - 5L\{e^{4t}\}$$
$$= 3\left\{\frac{1}{s-(-4)}\right\} - 5\left\{\frac{1}{s-4}\right\}$$
$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii $\sin 4t + \cos 4t$

$$L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$$
$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$
$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$$

iii $t^3 + 2t^2 - t + 4$

$$\Rightarrow L\{t^3 + 2t^2 - t + 4\} = L\{t^3\} + L\{2t^2\} - L\{t\} + L\{4\}$$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^{3+1}} + 2\left\{\frac{2!}{s^{2+1}}\right\} - \left\{\frac{1}{s^{1+1}}\right\} + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv $e^{-2t}\cos 5t$

$L\{\cos 5t\} = \frac{s}{s^2+a^2} = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$

Replacing s by a shift of e^{-2t} : $s+2$

$L\{e^{-2t}\cos 5t\} = \frac{s+2}{(s+2)^2+25}$

v. $t\sin 3t$

$L\{\sin 3t\} = \frac{a}{s^2+a^2} = \frac{3}{s^2+3^2}$

$F(s) = \frac{3}{s^2+9}$

$L\{t\sin 3t\} = -f'(s)$ or $-d/ds\,[f(s)]$

$f'(s)$ using quotient rule

$u = 3$ $\quad du = 0$

$v = s^2+9$ $\quad dv = 2s$

$\frac{v\,du/ds - u\,dv/ds}{v^2} = \frac{[s^2+9]\cdot 0 - 3\cdot[2s]}{[s^2+9]^2}$

$= \frac{-6s}{[s^2+9]^2}$

$-f'(s) = -1\left\{\frac{-6s}{[s^2+9]^2}\right\} = \frac{6s}{[s^2+9]^2}$

vi $\frac{e^{-t}-e^{-2t}}{t}$

$L\left\{\frac{e^{-t}-e^{-2t}}{t}\right\}$

$\lim_{t\to 0}\left\{\frac{e^{-t}-e^{-2t}}{t}\right\} = \frac{1-1}{0} = \frac{0}{0}$ => indeterminate

Applying L'Hopital's rule

$\lim_{t\to 0}\left\{\frac{-1e^{-t}-(-2)e^{-2t}}{1}\right\} = \left\{\frac{-1+2}{1}\right\} = \frac{1}{1} = 1$

$L\left\{\frac{f(t)}{t}\right\} = \int_{\sigma=s}^{\infty} F\sigma\, d\sigma$

$f\sigma = L\{f(t)\}$

$L\{f(t)\} = e^{-t} - e^{-2t} = \dfrac{1}{s+a} - \dfrac{1}{s+a}$

$= \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$f\sigma = \dfrac{1}{\sigma+1} - \dfrac{1}{\sigma+2}$

$\int_{\sigma=s}^{\infty} f\sigma \quad L\{f(t)/t\} = \int_{\sigma=s}^{\infty} \dfrac{1}{\sigma+1} - \dfrac{1}{\sigma+2} \, d\sigma$

$= \int_{\sigma=s}^{\infty} \dfrac{1}{\sigma+1} d\sigma - \int_{\sigma=s}^{\infty} \dfrac{1}{\sigma+2} d\sigma$

$= [\ln[\sigma+1] - \ln[s+2]]_s^{\infty}$

$= \left[\dfrac{\ln(\sigma+1)}{\sigma+2}\right]_s^{\infty} = \ln\left[\dfrac{\infty+1}{\infty+2} - \dfrac{s+1}{s+2}\right]$

$= -\ln\left[\dfrac{s+1}{s+2}\right] = \ln\left[\dfrac{s+2}{s+1}\right]$

vii $e^{4t} \cos 2t$

$L\{e^{4t}\cos 2t\} = e^{4t} \, L\{\cos 2t\}$

$L\{\cos 2t\} = \dfrac{s}{s^2+2^2} = \dfrac{s}{s^2+4}$

replacing s by a shift of e^{4t} : $s-4$

$L\{e^{4t}\cos 2t\} = \dfrac{s-4}{(s-4)^2+4}$

viii $t \sin 2t$

$= L\{t\sin 2t\} = \frac{-d}{ds}\{f(s)\}$

$F(s) = L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$f(s) = \frac{2}{s^2+4}$

$F'(s)$ using quotient rule

$u = 2$ $\quad du/ds = 0$

$v = s^2+4$ $\quad dv/ds = 2s$

$\frac{v\,du/ds - u\,dv/ds}{v^2}$

$\frac{-(s^2+4)\cdot 0 - 2\cdot(2s)}{[s^2+4]^2}$

$= \frac{-4s}{[s^2+4]^2}$

$\therefore L\{t\sin 2t\} = -f'(s)$

$= -1\cdot\left\{\frac{-4}{(s^2+4)^2}\right\} = \frac{4s}{(s^2+4)^2}$

ix t^3+4t^2+5

$L\{t^3+4t^2+5\} = L\{t^3\} + 4L\{t^2\} + L\{5\}$

$= \frac{3!}{s^{3+1}} + 4\left\{\frac{2!}{s^{2+1}}\right\} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

xi $e^{st}(t^2+4\}$

$\Rightarrow L\{t^2+4\} = L\{t^2\} + L\{4\}$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

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replacing s by a shift of $s-3$

$$L\{e^{3t}(t^2+4\} = \frac{2}{(s-3)^2} + \frac{4}{(s-3)}$$

xi $t^2 \cos t$

$$L\{t^2 \cos t\} = t^2 L\{\cos 3t\}$$

$$f(s) = L\{\cos t\} = \frac{s}{s^2+1^2} = \frac{s}{s^2+1}$$

$F'(s)$ using quotient rule

$u = s \qquad du/ds = 1$

$v = s^2+1 \qquad dv/ds = 2s$

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$\frac{[s^2+1].1 - 2s[s]}{(s^2+1)^2}$$

$$= \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{-s^2+1}{(s^2+1)^2} = \frac{s^2-1}{(s^2+1)^2}$$

$$-F''(s) = -\frac{d}{ds}\left\{\frac{s^2-1}{(s^2+1)^2}\right.$$

$u = s^2-1 \qquad du/ds = 2s$

$v = (s^2+1)^2 \qquad dv/ds = 4s\,(s^2+1)$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^3 - 4s]}{[(s^2+1)^2]}$$

$$= \frac{[2s^5-4s^3+2s] - [4s^5-4s]}{(s^2+1)^2}$$

$$= \frac{2s^5-4s^3+2s-4s^5+4s}{(s^2+1)^2}$$

$$= \frac{-2s^5-4s^3+6s}{(s^2+1)^2}$$

$$= \frac{-2s^5-4s^3+6s}{s^4+2s^2+1}$$

$$F''(s) = -\frac{d}{ds}\left\{\frac{-2s^5-4s^3+6s}{s^4+2s^2+1}\right\} = \frac{2s^5+4s^3-6s}{s^4+2s^2+1}$$

xii $\frac{\sinh 2t}{t}$

$= L\left\{ \frac{\sinh 2t}{t} \right\}$

3.i $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left\{ \frac{s-5}{(s-3)(s-4)} \right\} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

Assuming $s-4=0 \quad \therefore \quad s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s-3=0 \quad \therefore \quad s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{(s-3)} + \frac{-1}{(s-4)}$

$= \frac{2}{(s-3)} - \frac{1}{s-4}$

$2\left\{ \frac{1}{s-3} \right\} - \left\{ \frac{1}{s-4} \right\}$

$= 2e^{3t} - e^{4t}$

iii $\frac{2s-6}{(s-2)(s-4)}$

$L^{-1}\left\{ \frac{2s-6}{(s-2)(s-4)} \right\}$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s = 4$

$$2(4)-6 = A(4-4) + B(4-2)$$

$$8-6 = A(0) + B(2)$$

$$2 = 2B$$

$$B = 1$$

Assuming $s = 2$

$$2(2)-6 = A(2-4) + B(2-2)$$

$$4-6 = A(-2) + 0$$

$$-2 = -2A$$

$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}.$$

iii

$$\frac{5s-8}{s(s-4)}$$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

Assuming $s = 4$

$$5(4)-8 = 4B$$

$$12 = 4B$$

$$B = 3$$

Assuming $s = 0$

$$5(0)-8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left\{\frac{1}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$

iv. $\dfrac{s^2 - 3s - 4}{(s-3)(s-1)^2}$

$$F(s) = \frac{A}{s-3} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$$

$$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

if $s - 1 = 0 \quad \therefore \quad s = 1$

$$1^2 - 3(1) - 4 = A(1-1)^2 + B(1-3)(1-1) + C(1-3)$$

$$1 - 3 - 4 = A(0) + B(0) + C(-2)$$

$$-6 = -2C$$

$$C = 3$$

if $s - 3 = 0 \quad \therefore \quad s = 3$

$$3^2 - 3(3) - 4 = A(3-1)^2 + B(3-3)(3-1) + C(3-3)$$

$$9 - 9 - 4 = A(2)^2 + B(0) + C(0)$$

$$-4 = 4A$$

$$A = -1$$

s^2 : $1 = A + B$

$$1 = -1 + B$$

$$1 + 1 = B$$

$$B = 2$$

$$F(s) = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$F(t) = -e^{3t} + 2e^t + 3te^t$$

$$e^t[2 + 3t] - e^{3t}.$$

iv. $\dfrac{s-5}{s^2+4s+20}$

$$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} =$$

$$F(s) = \frac{s\ \cancel{}}{(s^2+2)^2+16} - \frac{5}{(s^2+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7 \times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right\}$$