15/ENG 03/025

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ENG 381

1) $(1-x^2)d^2y/dx^2 - 2x\,dy/dx + 2y = 0$ -(1)

This can be written as

$(1-x^2)y'' - 2xy' + 2y = 0$

let $w_1 = (1-x^2)y''$

solving using leibnitz

$u = y''$, $u^n = y^{(n+2)}$

$v = 1-x^2$, $v' = -2x$, $v'' = -2$, $v''' = 0$

$w_1^n = y^{n+2}(1-x^2) + ny^{(n+2-1)}\cdot(2x) + \frac{n(n-1)}{2!}y^{(n+2-2)}\cdot(-2) + \frac{n(n-1)(n-2)}{3!}y^{(n+2-3)}(0)$

$w_1^n = (1-x^2)y(n+2) - 2xny^{(n+1)} - n(n-1)y^n$

let $w_2 = -2xy'$

$u = y'$, $u^n = y^{(n+1)}$

$v = -2x$, $v' = -2$, $v'' = 0$

$w_2^n = y^{(n+1)}\cdot(-2x) + ny^{(n+1-1)}\cdot(-2) + n(n-1)y^{(n+1-2)}\cdot 0$

$= -2xy^{n+1} - 2ny^n$

let $w_3 = 2y$

$u = y$, $u^{(n)} = y^{(n)}$

$v = 2$, $v' = 0$

$w_3^n = 2y^n$

$y^n = w_1^n + w_2^n + w_3^n$

$y^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} - 2x(n+1)y^{n+1} + (-n^2+n-2n+2)y^n$

$y^n = (1-x^2)y^{n+2} - 2x(n+1)y^{n+1} + (-n^2-n+2)y^n$

at $x=0$ and $y^n = 0$

$0 = (1-0^2)y^{n+2} - 2(0)(n+1)y^{n+1} + (-n^2-n+2)y^n$

$= y^{n+2} + (-n^2-n+2)y^n$

$y^{n+2} = (n^2+n-2)y^n$

$n \geq 0$, $n = 0, 1, 2, 3, 4, 5 \ldots$

at $n=0$

$y^2 = -2(y)_0$

at $n = 1$

$y^3 = (1+1-2)y^3$

$y^3 = 0$

at $n = 2$

$y^4 = (2^2 + 2 - 2)y^2$

$y^4 = 4y^2 = 4[-2(y_0)] = -8[y_0]$

at $n = 3$

$y^5 = (3^2 + 3 - 2)y^3$

$= (10y^3) = 10(y^3) = 0$

at $n = 4$

$y^6 = (4^2 + 4 - 2)y^4$

$= (16 + 4 - 2)y^4 = 18(y^4)_0 = 18(-8y_0) = -144(y)_0$

at $n = 5$

$y^7 = (5^2 + 5 - 2)y^5$

$= (28)y^5 = 28y^3 y^5 = 0$

Recall that from maclaurin's

$$y(x) = (y_0) + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^4)_0 + \cdots$$

substituting

$$= (y)_0 + x(y')_0 + \frac{x^2}{2!}(-2(y)_0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8(y_0)) + \frac{x^5}{5!}(0) +$$

$$\frac{x^6}{6!}(-144(y)_0) + \frac{x^7}{7!}(0)$$

$$y(x) = y_0 + xy'_0 - \frac{2x^2}{2}y_0 - \frac{8x^4}{4!} - \frac{144x^6}{6!}y_0$$

$$= y_0 + xy'_0 - x^2y_0 - \frac{8x^4y_0}{24} - \frac{144}{720}x^6y_0$$

2) solve $3e^{-4t} - 5e^{4t} = f(t)$

i) $f(s) = L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left(\frac{1}{s+4}\right) - 5\left(\frac{1}{s-4}\right)$

$= \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} = \frac{3s-12-5s-20}{(s-4)(s+4)}$

$= \frac{-2s-32}{s^2-16}$

ii) $\sin 4t + \cos 4t = f(t)$

$F(s) = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4(s^2+16) + s(s^2+16)}{(s^2+16)^2} = \frac{4s^2+64+s^3+16s}{(s^2+16)^2} = \frac{s^3+4s^2+16s+64}{(s^2+16)^2}$

iii) $t^3 - 2t^2 - t + 4 = f(t)$

$F(s) = L(t^3) + L(2t^3) - L[t] + L[4]$

$= \frac{3!}{s^4} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6+4s-s^2+4s^3}{s^4}$

iv) $e^{-2t}\cos 5t = f(t)$

$F(s) = L[e^{-2t}\cos 5t]$

$= \frac{s+3}{(s+5)^2+5^2} = \frac{5+s}{s^2+10s+25+25} \neq \frac{s+5}{s^2+10s+50}$

v) $t \sin 3t = f(t)$

$F(s) = L[t \sin 3t]$

$= \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$

using quotient rule

$F(s) = -1\left[\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2}\right]$

$= -\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

vi) $\frac{e^{-t} - e^{-2t}}{t} = f(t)$

$F(s) = L\left[\frac{1}{t}\left(e^{-t} - e^{-2t}\right)\right]$

$= \int \left(\frac{1}{s+1} - \frac{1}{s+2}\right)$

$= \int \frac{s+2-s-1}{(s+1)(s+2)}$

$= \int \frac{1}{s^2+3s+2}\, ds$

$F(s) = \ln(s^2+3s+2)$

vii) $e^{4t}\cos 2t = f(t)$

$F(s) = L[e^{4t}\cos 2t]$

$= \frac{s-4}{(s-4)^2+4^2}$

$= \frac{s-4}{s^2-8s+16+16}$

$= \frac{s-4}{s^2-82+32}$

(viii) $t\sin 2t = f(t)$

$F(s) = L[t\sin 2t]$

$= -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

Using quotient rule

$F(s) = -\left[\frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

(ix) $t^3 + 4t^2 + 5 = f(t)$

$F(s) = L[t^3 + 4t^2 + 5]$

$= L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + 5/s$

$= 6/s^4 + \frac{8}{s^3} + 5/s$

$= \frac{6 + 8s + 5s^3}{s^4} = \frac{5s^3 + 8s + 6}{s^4}$

(x) $e^{3t}(t^2+4) = f(t)$

$= e^{3t}t^2 + 4e^{3t} = f(t)$

$F(s) = L[e^{3t}t^2] + L[4e^{3t}]$

$= \frac{2!}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{2(s-3) + 4(s-3)^2}{(s-3)^3}$

$= \frac{2s - 6 + 4(s^2 - 6s + 9)}{(s-3)^3}$

$= \frac{2s - 6 + 4s^2 - 24s + 36}{(s-3)^3}$

$= \frac{4s^2 - 26s + 30}{(s-3)^3} = \frac{4s^2 - 26s + 30}{(s-3)^3}$

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(3) Convert the following functions to the (t) dom

$$\frac{S-5}{(S-3)(S-4)}$$

Solution

$$\frac{S-5}{(S-3)(S-4)} = \frac{A}{S-4} + \frac{B}{S-3}$$

at $S=4$

$4-5 = A(4-3)$

$-1 = A \quad \therefore A = -1$

at $S=3$

$3-5 = B(3-4)$

$-2 = -B$

$B = 2$

$$\frac{S-5}{(S-3)(S-4)} = \frac{-1}{S-4} + \frac{2}{S-3}$$

$F(t) = -e^{4t} + 2e^{3t}$

$= \underline{2e^{3t} - e^{4t}}$

(ii) $$\frac{2S-6}{(S-2)(S-4)}$$

Solution

$$= \frac{A}{S-2} + \frac{B}{S-4}$$

at $S=2$

$2(2)-6 = A(2-4)$

$-2 = -2A$

$A = 1$

at $S=4$

$2(4)-6 = B(4-2)$

$2 = 2B$

$B = 1$

$$\frac{2S-6}{(S-2)(S-4)} = \frac{1}{S-2} + \frac{1}{S-4}$$

taking inverse Laplace

$f(t) = \underline{e^{2t} + e^{4t}}$

(iii) $\frac{5s-8}{s(s-4)}$

Solution

$\equiv \frac{A}{s} + \frac{B}{s-4}$

taking limits

at $s=0$

$5(0)-8 = A(0-4)$

$-8 = -4A$

$A = 2$

at $s=4$

$5(4)-8 = B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

$\frac{5s-8}{s(s-4)} \equiv \frac{2}{s} + \frac{3}{s-4}$

taking inverse laplace

$f(t) = 2 + 3e^{4t} = 2u(t) + 3e^{4t}$

(iv) $\frac{s^3-3s-4}{(s-3)(s-1)^2}$

Solution

$\frac{s^3-3s-4}{(s-3)(s-1)^2} \equiv \frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$

taking limits

at $A=3$

$3^3-3(3)-4 = A(3-1)^2$

$27-9-4 = 4A$

$14 = 4A$

$A = 7/2$

for B

at $s=1$ $\quad \dfrac{2s^3-9s^2+13}{(s-3)^2}$

$$= \frac{2(1)^3-9(1)^2+13}{(1-3)^2} = \frac{2-9+13}{(-2)^2} = \frac{6}{4} = \frac{3}{2}$$

for C

at $s=1$ $\quad = \dfrac{s^3-3s-4}{(s-3)} = \dfrac{(1)^3-3(1)-4}{(1-3)} = \dfrac{1-3-4}{-2}$

$$= \frac{-6}{-2} = 3$$

$$F(t) = L^{-1}\left[\frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{s-1}\right]$$

$$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}e^{t} + 3e^{t} //$$

(v) $\dfrac{s-5}{s^2+4s+20}$

~~Solution~~

$$\cancel{= \frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)} = \frac{As+B}{(s^2+4s+20)}}$$

$$\cancel{s-5 = As+B}$$

$$\cancel{A = 1 \text{ and } B = -5}$$

$$\cancel{F(s) = s-5}$$

(v) $= \dfrac{A}{(s+2-j4)} + \dfrac{B}{(s+2+4j)} = \dfrac{s-5}{(s+2-j4)(s+2+4j)}$

$$A|_{s=-2+4j} = \frac{(-2+4j)-5}{-2+4j+2+4j} = \frac{-7+4j}{8j} \times \frac{j}{j}$$

$$= \frac{-7j-4}{-8} = \frac{7j+4}{8}$$

$$B|_{s=-2-4j} = \frac{-2-4j-5}{-2-4j+2-4j} = \frac{-7j+4}{8}$$

$$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right) //$$