OJUMO ADEGBENRO JOHN
15/ENG05/010
CHEMICAL ENGINEERING
ENG381 ASSIGNMENT IV

1) $(1-x^2)\,d^2y/dx^2 - 2x\,dy/dx + 2y = 0$

$(1-x^2)\,y^{(2)} - 2x\,y^{(1)} + 2y^{(0)} = 0$

Let $w_1 = (1-x^2)\,y^{(2)}$

$u = y^{(2)}$ $\quad v = (1-x^2)$

$u^{(n)} = y^{(n+2)}$ $\quad v' = -2x$

$u^{n-1} = y^{(n+1)}$ $\quad v'' = -2$

$u^{n-2} = y^n$

$w_1^{(n)} = (1-x^2)\,y^{(n+2)} + n\,(y^{(n+1)})(-2x) + n(n-1)\,y^n(-2)$

$w_1^{(n)} = (1-x^2)\,y^{(n+2)} - 2xn\,y^{(n+1)} - (n^2-n)\,y^{(n)}$

$w_2 = -xy^{(1)}$

$u = y^{(1)}$ $\quad v = x$

$u^n = y^{n+1}$ $\quad v' = 1$

$u^{n-1} = y^{(n)}$

$w_2^{(n)} = xy^{(n+1)} + ny^{(n)}$

$w_3 = y^{(0)}$

$w_3^{(n)} = y^n$

$\therefore (1-x^2)\,y^{(n+2)} - 2xn\,(y^{n+1}) - (n^2-n)\,y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)} = 0$

$y^{(n+2)}(1-x^2) - 2xy^{(n+1)}(n+1) - y^{(n)}(n^2-n+2n-2) = 0$

$y^{(n+2)}(1-x^2)\ 2x - y^{(n+1)}(n+1) - y^{(n)}(n^2+n-2) = 0$

Let $x = 0$

$y^{(n+2)} - y^{(n)}(n^2+n-2) = 0$

$y^{(n+2)} = y^{(n)}(n^2+n-2)$

$n=0 \quad y^{(2)} = y^{(0)}(-2)$

$n=1 \quad y^3 = y^1(0) = 0$

$n=2 \quad y^4 = y^2(4) = -8(y^1)$

$n=3 \quad y^5 = y^3(10) = 0$

$n=4 \quad y^6 = y^4(18) = -8(18)\,y^{(1)}$

$y = y^{(0)} + xy^{(1)} + \frac{x^2}{2!}\,y^{(0)}(-2) + 0 + \frac{x^4}{4!}(8y^1) + 0\ (-8(18)\,y^{(1)})\frac{x^6}{6!}$

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$$y = y^{(0)} + xy^{(1)} - x^2 y^{(0)} - \frac{x^4}{3} y^{(0)} - \frac{x^6}{5} y^{(0)} \ldots$$

$$y = xy^{(1)} + y^{(0)}(1 - x^2 - x^4/3 - x^6/5)$$

2. (i) $x(t) = 3e^{-4t} - 5e^{4t}$

$x(s) = L[x(t)] = 3L[e^{-4t}] - 5L[e^{4t}]$

$$x(s) = \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $x(t) = \sin 4t + \cos 4t$

$$x(s) = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

(iii) $x(t) = t^3 + 2t^2 - t + 4$

$$x(s) = \frac{3!}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$x(s) = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $x(t) = e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+5^2}$$

$$x(s) = \frac{s+2}{(s+2)^2 + 25}$$

(v) $x(t) = t \sin 3t$

$$L[\sin 3t] = \frac{3}{s^2+9}$$

$$x(s) = -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$x(s) = -\frac{d}{ds}[3(s^2+9)^{-1}]$$

$$x(s) = \frac{6s}{(s^2+9)^2}$$

vi] $\dfrac{e^{-t} - e^{-2t}}{t}$

$\lim_{t \to 0} \dfrac{e^{-0} - e^{-0}}{0}$ = undefined

using L' hopital's rule

$\lim_{t \to 0} \dfrac{-e^{-t} + 2e^{-2t}}{1}$ = real ans

$L[e^{-t} - e^{-2t}] = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$L\left[\dfrac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma = s}^{\infty} \left(\dfrac{1}{\sigma+1} - \dfrac{1}{\sigma+2}\right)$

$= \ln(\sigma+1) \Big|_s^{\infty} - \ln(\sigma+2) \Big|_s^{\infty}$

$= \left[\ln\left(\dfrac{\sigma+1}{\sigma+2}\right)\right]_s^{\infty}$

$= \ln \dfrac{\infty+1}{\infty+2} - \ln \dfrac{s+1}{s+2}$

$= \ln 1 - \ln \dfrac{s+1}{s+2}$

$= 0 - \ln \dfrac{s+1}{s+2}$

$= -\ln \dfrac{s+1}{s+2}$

$= \ln \dfrac{s+2}{s+1}$

vii] $x(t) = e^{4t} \cos 2t$

$L[\cos 2t] = \dfrac{s}{s^2+4}$

$x(s) = \dfrac{s-4}{(s-4)^2+4}$

viii] $x(t) = t \sin 2t$

$L[\sin 2t] = \dfrac{2}{s^2+4}$

$x(s) = -\dfrac{d}{ds}\left(\dfrac{2}{s^2+4}\right)$

$x(s) = \frac{4s}{(s^2+4)^2}$

ix] $x(t) = t^3 + 4t^2 + 5$

$x(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

2. x] $x(t) = e^{3t}(t^2+4)$

$L[t^2+4] = \frac{2}{s^3} + \frac{4}{s}$

$x(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

xi] $x(t) = t^2 \cos t$

$L[\cos t] = \frac{s}{s^2+1}$

$x(s) = \frac{d^2}{ds^2}\left(\frac{s}{s+1}\right)$

$x(s) = \frac{d}{ds}\left(\frac{d}{ds}\left(\frac{s}{s+1}\right)\right)$

$x(s) = \frac{d}{ds}\left(\frac{1}{s+1} - \frac{s}{(s+1)^2}\right)$

$x(s) = \frac{-1}{(s+1)^2} - \frac{1}{(s+1)^2} + \frac{2s}{(s+1)^3}$

$x(s) = \frac{2s}{(s+1)^3} - \frac{2}{(s+1)^2}$

xii] $x(t) = \frac{\sinh 2t}{t}$

$L[\sinh 2t] = \frac{2}{s^2-4}$

$x(s) = \int_s^\infty \frac{2}{s^2-4}$

$x(s) = \int_s^\infty \frac{2}{(s-2)(s+2)}$

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$$\frac{2}{(s-2)(s+2)} = \frac{A}{s-2} + \frac{B}{s+2}$$

$$2 = A(s+2) + B(s-2)$$

$$2 = (A+B)s + 2A - 2B$$

Comparing both sides

$$2A - 2B = 2$$

$$A - B = 1 \longrightarrow (1)$$

$$A + B = 0 \longrightarrow (2)$$

$$2A = 1$$

$$A = 1/2$$

Subst into (1)

$$1/2 - B = 1$$

$$B = -1/2$$

$$\therefore x(s) = \int_s^\infty \frac{1}{2(\sigma-2)} - \frac{1}{2(\sigma+2)}$$

$$\cancel{x(s) = \frac{1}{2}\ln(\sigma-2)|_s^\infty}$$

$$x(s) = 1/2 \ln(\sigma-2|_s^\infty - 1/2 \ln(\sigma+2)|_s^\infty$$

$$x(s) = 1/2 \left[\ln \frac{\sigma-2}{\sigma+2}\right]_s^\infty$$

$$x(s) = 1/2 \left(\ln \frac{\infty-2}{\infty+2} - \ln s-2/s+2\right).$$

$$x(s) = -1/2 \ln s-2/s+2.$$

$$x(s) = \ln \sqrt{\frac{s+2}{s-2}}$$

3) $$x(s) = \frac{s-5}{(s-3)(s-4)}$$

$$x(s) = \frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

$$s-5 = (A+B)s - 4A - 3B$$

Comparing both sides

$A+B=1$

$-4A-3B=-5 \quad -2$

from 1 $\quad A=1-B$

So, $-4(1-B)-3B=-5$

$-4+4B-3B=-5$

$B-4=-5$

$B=-1$

$A=1-(-1)$

$A=2$

$\therefore x(s)=\frac{2}{s-3}-\frac{1}{s-4}$

$x(t)=2e^{3t}-e^{4t}$

iv] $\frac{2s-6}{(s-2)(s-4)}=\frac{A}{s-2}+\frac{B}{s-4}$

$2s-6=A(s-4)+B(s-2)$

$2s-6=(A+B)s-4A-2B$

$A+B=2$

$A=2-B$

$-4A-2B=-6$

$-4(2-B)-2B=-6$

$-8+4B-2B=-6$

$2B \cancel{+4B} -8=-6$

$2B=2$

$B=1$

$A=2-1$

$A=1$

$x(s)=\frac{1}{s-2}+\frac{1}{s-4}$

$x(t)=e^{2t}+e^{4t}$

m] $x(s) = \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s - 8 = A(s-4) + Bs$

$5s - 8 = (A+B)s - 4A$

$-8 = -4A$

$A = 2$

$A + B = 5$

$B = 5 - A$

$B = 5 - 2$

$B = 3$

$x(s) = \frac{2}{s} + \frac{3}{s-4}$

$x(t) = 2 + 3e^{4t}$

n] $x(s) = \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$s^2 - 3s - 4 = A(s-1)^2 + B(s-1)(s-3) + C(s-3)$

Let $s = 1$

$1 - 3 - 4 = -2C$

$-6 = -2C$

$C = 3$

Let $s = 3$

$9 - 9 - 4 = 4A$

$A = -1$

$s^2 - 3s = -1(s-1)^2 + B(s-1)(s-3) + 3(s-3)$

$s^2 - 3s - 4 = -s^2 + 2s - 1 + B(s^2 - 3s - s + 3) + 3s - 9$

$s^2 - 3s - 4 = -s^2 + 2s - 1 + Bs^2 - 4Bs + 3B + 3s - 9$

Comparing both sides

$B - 1 = 1$

$B = 2$

$\therefore x(s) = -1/s-3 + 2/s-1 + 3/(s-1)^2$

$x(t) = -e^{3t} + 2e^{t} + 3te^{t}$

$x(t) = 3te^{t} + 2t^{t} - e^{3t}$

v) $x(s) = \dfrac{s-5}{s^2+4s+20} = \dfrac{s-5}{s^2+4s+16+4}$

$= \dfrac{s-5}{(s+2)^2+4}$

$= \dfrac{s-5+2-2}{(s+2)^2+4}$

$= \dfrac{s+2}{(s+2)^2+4} - \dfrac{7}{(s+2)^2+4}$

$= \dfrac{s+2}{(s+2)^2+2^2} = \dfrac{7}{(s+4)^2+2^2}$

$x(t) = e^{-2t}\cos 2t - 7e^{-2t}$.