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15/ENG04/055
Elect-Elect
ENG 381

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

General equation

$W^n = U^n V^0 + nx^{n-1}V' + \frac{n(n-1)U^{n-2}V''}{2!} + \frac{n(n-1)(n-2)U^{n-3}V'''}{3!}$

* for $(1-x^2)y''$ let $u = y''$, $u' = y'''$, $u^n = y^{n+2}$

$V = V^0 = 1-x^2$, $V' = -2x$, $V'' = -2$, $V''' = 0$

for xy'

$u = y'$, $u = y''$, $u'' = y'''$, $u^n = y^{n+1}$

$V = V^0 = x$, $V' = 1$, $V'' = 0$

$W_2^n = x \cdot y^{n+1} + n \cdot y^n \cdot 1$

$= xy^{n+1} + ny^n$

for $y = y^n = +W_3$

$W^n = W_1^n + W_2^n + W_3^n$

$W^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2(xy^{n+1} + ny^n)$

$D = (1-x^2)y^{n+2} - 2xny^{n+1} - n\,y^{n+2}y^n$

if $x = 0$

$D = y^{n+2} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^{n+2} - (n^2 - n + 2n - 2)y^n$

$y^{n+2} = (n^2+n-2)y^n$

for $n=1$

$y^3 = y''' = (1^2+1-2)y' = 0$

for $n=2$

$y^4 = y^{iv} = (2^2+2-2)y'' = 4y''$

for $n=3$

$y^5 = y^v = (3^2+3-2)y''' = 10y''' = 0$

for $n=4$

$y^6 = y^{vi} = (4^2+4-2)y^{iv} = 18y^{iv} = 18(4y'')$

for $n=5$

$y^n = y^{vii} = (5^2+5-2)y^v = 28y^v$ —①

for $n=8$

$y^8 = y^{viii} = (6^2+6-2)y^{vi} = 40y^{vi} = (40 \times 18 \times 4)y''$

$y = 1 + f(x) + y'f'(0) + y'' \frac{f'(x)}{2!} +$

$y = 1y^0 + y' + \frac{y''}{2!} + \frac{4y''}{4!} + \frac{18y^4}{6!}(4) + \frac{40(18 \times 4)y^4}{8!}$

$y = 1 + y^0 + y' + y'' \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{10} + \frac{1}{14}\right]$

2i) $L\left[3e^{-4t} - 5e^{4t}\right] = \frac{3}{s+4} - \frac{5}{s-4}$

(ii) $L\left[\sin 4t + \cos 4t\right) = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{s^2+16}$

(iii) $\left[t^3 + 2t^2 - t + 4\right] = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1!}{s^{1+1}} + \frac{4}{s}$

$= \frac{1}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $(e^{-2t}\cos 5t) = \frac{s+2}{(s+2)^2+5^2} = \frac{s+2}{s^2+4s+29}$

(v) $(t\sin 3t) = (-1)^2 \frac{d^n}{ds^n}[f(t)) = -1 \frac{d}{ds}\left[1, \frac{3}{s^2+3^2}\right]$ $d=0$ $dv = 2s$

$= -\left[\frac{-6r}{(s^2+9)^2}\right) = \frac{6s}{(s^2+9)^2}$

(vi) $\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \left[\frac{\frac{1}{s+1} - \frac{1}{s+2}}{\frac{1}{s^2}}\right] = \left[\frac{1}{s+1} - \frac{1}{s+2}\right] \cdot s^2$

$= \frac{s^2}{(s+1)(s+2)}$

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(Q i) $(e^{4t}\cos 2t) = \frac{s}{(s-4)^2+2^2} = \frac{s}{s^2-8s+20}$

(Q iii) $[t\sin 2t] = -1\cdot\frac{d}{ds}\left(\frac{2}{s^2+2^2}\right)$ $du=0$ $dv=2s$

$= -1\cdot\left[\frac{-4s}{(s^2+2^2)^2}\right]$ $du=0$ $dv=2s$ $= \frac{4s}{(s^2+4)^2}$

(i v) $t^3+4t^2+5 = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $e^{3t}(t^2+4) = t^2e^{3t}+4e^{3t} = \frac{2!}{(s-3)^{1+1}} + \frac{4}{s-3}$

$= \frac{2}{(s-3)^2} + \frac{4}{s-3}$

(xi) $t^2\cos t = (-1)^2\cdot\frac{d^2}{ds^2}\left(\frac{s}{s^2+1}\right) = \frac{d}{ds}\left[\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right]$ $du=1$ $dv=2s$

$\frac{d}{ds}\left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right) = \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$ $du=2s$ $dv=4s^3+4s$

$= \left[\frac{-2s^5-4s^3-2s-4s+4s^5}{((s^2+1)^2)^2}\right] = \left[\frac{2s^5-4s^3-6s}{(s^2+1)^4}\right]$

(x) $\frac{\sinh 2t}{t} = \frac{1}{2}\ln(s^2-4)-\ln s$

(3 i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

$3-5 = A(3-4) \Rightarrow A=2$

$4-5 = B(4-3) \Rightarrow B=-1$

$L^{-1}\left\{\frac{2}{s-3} - \frac{1}{s-4}\right\} = 2e^{3t}-e^{4t}$

(5 ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$2(2)-6 = A(2-4) \Rightarrow A=1$

$2(4)-6 = B(4-2) \Rightarrow B=1$

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$$L^{-1}\left(\frac{1}{s-2}+\frac{1}{s-4}\right)=e^{2t}+e^{4t}$$

(i) $\frac{5s-8}{s(s-4)}=\frac{A}{s}+\frac{B}{s-4}$

$5s-8=A(s-4)+Bs$

$5(0)-8=A(0-4)\Rightarrow A=2$

$5(4)-8=B(4)\Rightarrow B=3$

$L^{-1}\left[\frac{2}{s}+\frac{3}{s-4}\right)=2u(t)+3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}=\frac{A}{s-3}+\frac{B}{s-1}+\frac{C}{(s-1)^2}$

$s^2-3s-4=A(s-1)^2+B(s-1)^2+B(s-3)(s-1)+C(s-3)$

$1^2-3(1)-4=C(1-3)\Rightarrow C=3$

$s^2-3s-4=(s^2-2s+1)A+(s^2-4s+3)B+(s-3)C$

$=-2A-4B+C=-3$

$-2(-1)-4(B)+3=-3$

$-4B=-3-3-2$

$-4B=-3-3-2$

$B=2$

$L^{-1}\left(\frac{-1}{s-3}+\frac{2}{s-1}+\frac{3}{(s-1)^2}\right)$

$=-e^{3t}+2e^{t}+3te^{t}$

(v) $\frac{s-5}{s^2+4s+20}$

$=\frac{s-5}{s^2+4s+4+16}$

$=\frac{s-5}{(s+2)^2+4}$

$=\frac{s-5}{(s+2)^2+4}=(e^{2t}-7)\cos 4t$