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MECHATRONICS

① $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ ——— (i)

This can be written as

$(1-x^2)y'' - 2xy' + 2y = 0$ ——— (ii)

let $w_1 = (1-x^2)y''$

solving using Leibnitz

$u = y''$ $\quad v = 1-x^2$

$u^n = y^{n+2}$ $\quad v' = -2x$

$v'' = -2$

$v''' = 0$

$w_1^n = y^{n+2}(1-x^2) + ny^{(n+2-1)}(-2x) + \frac{n(n-1)}{2!}y^{(n+2-2)}(-2) + n(n-1)(n-2)y^{(n+2-3)} + 0$

$w_1^n = (1-x^2)y^{(n+2)} - 2xy^{(n+1)} - 2n(n-1)y^n$ ——— (iii)

let $w_2 = -2xy$

$u = y'$ $\quad v = -2x$

$u_n = y^{(n+1)}$ $\quad v' = -2$

$v'' = 0$

$w_2^n = y^{(n+1)}\cdot(-2x) + ny^{(n+1-1)}\cdot(-2) + n(n-1)y^{(n+1-2)}\cdot 0$

$= 2xy^{n+1} - 2ny^n$ ——— (iv)

let $w_3 = 2y$

$u = y$ $\quad v = 2$

$u^n = y^n$ $\quad v' = 0$

$y_3^n = 2y^n$

$y^n = w_1^n + w_2^n + w_3^n$

$y^n = (1-x^2)y^{n+2} - 2xny^{n+1} - 2n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+1} - 2x(n+1)y^{n+1} + (-n^2 + n - 2n + 2)y^n$

$y^n = (1-x^2)y^{n+2} - 2x(n+1)y^{n+1} + (-n^2 - n + 2)y^n$

at $x = 0$ and $y^n = 0$

$y^n = 0 = (1-0^2)y^{n+2} - 2(0)(n-1)y^{n+1} + (-n^2 - n + 2)y^n$

$= y^{n+2} + (-n^2 - n + 2)y^n$

$$y^{n+2} = (n^2+n-2)y^n$$

$n=0$

$n=0, 1, 2, 3, 4, 5$

* at $n=0$

$y^2 = -2(y)_0$

* at $n=1$

$y^3 = (1+1-2)y^3$

$y^3 = 0$

* at $n=2$

$y^4 = (2^2+2-2)y^2$

$y^4 = 4y^2 = 4(-2(y_0)) = -8y_0$

* at $n=3$

$y^5 = (3^2+3-2)y^3$

$= (10y^5) = 10(y^5) = 0$

* at $n=4$

$y^6 = (4^2+4-2)y^4$

$= (16+4-2)y^4 = 18(y^4)_0 = 18(-8y_0) = -144(y_0)$

* at $n=5$

$y^7 = (5^2+5-2)y^5$

$= (28)y^5 = 28y^0y^5 = 0$

Recall that from maclaurins

$$y(x) = (y_0) + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0 +$$

substituting

$$= (y_0) + x(y')_0 + \frac{x^2}{2!}(-2(y)_0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-8(y_0)) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(-144(y_0))$$

$$+ \frac{x^7}{7!}(0)$$

$$y(x) = y_0 + xy_0' - \frac{2x^2}{2!}y_0 + \frac{8x^4}{4!}y_0 - \frac{144x^6}{6!}y_0$$

$$= y_0 + xy_0' - x^2y_0 - \frac{8}{24}x^4y_0 - \frac{144}{720}x^6y_0$$

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(i) Solve $3e^{-4t} - 5e^{4t} = f(t)$

for $\bar{f}(s) = L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left(\frac{1}{s+4}\right) - 5\left(\frac{1}{s-4}\right)$

$= \frac{3(s-4)-5(s+4)}{(s+4)(s-4)} \quad = \frac{3s-12-5s-20}{(s-4)(s+4)}$

$= \frac{-2s-32}{s^2-16}$

(ii) $\sin 4t + \cos 4t = f(t)$

$\bar{f}(s) = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{4(s^2+16)+s(s^2+16)}{(s^2+16)^2} \quad = \frac{4s^2+64+s^3+16s}{(s^2+16)^2}$

$= \frac{s^3+4s^2+16s+64}{(s^2+16)^2}$

(iii) $t^3 + 2t^2 - t + 4 = f(t)$

$\bar{f}(s) = L[t^3] + L[2t^2] - L[t] + L[4]$

$= \frac{3!}{s^4} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6+4s-s^2+4s^3}{s^4}$

$= \frac{4s^3-s^2+4s+6}{s^4}$

(iv) $e^{-2t}\cos 5t = f(t)$

$\bar{f}(s) = L[e^{-2t}\cos 5t]$

$= \frac{s+5}{(s+5)^2+5^2}$

$= \frac{s+5}{s^2+10s+25+25} \quad = \frac{s+5}{s^2+10s+50}$

(x) $t\sin 3t = f(t)$

$f(s) = L[t\sin 3t]$

$= \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$

Using quotient rule

$f(s) = -1\left[\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2}\right]$

$= -\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

(VI) $\frac{e^{-t} - e^{-2t}}{t} = f(t)$

$f(s) = L\left[\frac{1}{t}\left[e^{-t} - e^{-2t}\right]\right]$

$= \int\left(1/s+1 - 1/s+2\right)$

$= \int \frac{s+2-s-1}{(s+1)(s+2)} = \int \frac{1}{s^2+3s+2}\, ds$

$f(s) = \ln\left(s^2+3s+2\right)$

(xiv) $e^{4t}\cos 2t = f(t)$

$f(s) = L\left[e^{4t}\cos 2t\right]$

$= \frac{s-4}{(s-4)^2+4^2} = \frac{s-4}{s^2-8s+16+16}$

$= \frac{s-4}{s^2-8s+32}$

(xiii) $t\sin 2t = f(t)$

$f(s) = L[t\sin 2t]$

$= \frac{-d}{ds}\left[\frac{2}{s^2+4}\right]$

Using quotient rule

$f(s) = -\left[\frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}\right] = \frac{4s}{(s^2+4)^2}$

(ix) $t^3 + 4t^2 + 5 = f(t)$

$\bar{f}(s) = L[t^3 + 4t^2 + 5]$

$= L[t^3] + L[4t^2] + L[5]$

$= \frac{3!}{s^4} + 4\left[\frac{2!}{s^3}\right] + 5/s$

$= 6/s^4 + 8/s^3 + 5/s$

$= \frac{6 + 8s + 5s^3}{s^4} = \frac{5s^3 + 8s + 6}{s^4}$

(8) $e^{3t}(t^2+4) = f(t)$

$= e^{3t}t^2 + 4e^{3t} = f(t)$

$\bar{f}(s) = L[e^{3t}t^2] + L[4e^{3t}]$

$= \frac{2!}{(s-3)^3} + \frac{4}{(s-3)}$

$= \frac{2(s-3) + 4(s-3)^2}{(s-3)^3}$

$= \frac{2s - 6 + 4(s^2 - 6s + 9)}{(s-3)^3}$

$= \frac{2s - 6 + 4s^2 - 24s + 36}{(s-3)^3}$

$= \frac{4s^2 - 26s + 30}{(s-3)^3} = \frac{4s^2 - 26s + 30}{(s-3)^3}$

(3) Convert the following functions to the (t) domain

$\frac{s-5}{(s-3)(s-4)}$

Solution

$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-4} + \frac{B}{s-3}$

at $s = 4$

$4 - 5 = A(4-3)$

$-1 = A \therefore A = -1$

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at $s = 3$

$3 - 5 = B(3 - 4)$

$-2 = -B$

$B = 2$

$$\frac{s-5}{(s-3)(s-4)} = \frac{-1}{s-4} + \frac{2}{s-3}$$

$f(t) = -e^{4t} + 2e^{3t}$

$= 2e^{3t} - e^{4t}$

(v) $\frac{2s-6}{(s-2)(s-4)}$

Solution

$$= \frac{A}{s-2} + \frac{B}{s-4}$$

at $s = 2$

$2(2) - 6 = A(2-4)$

$-2 = -2A$

$A = 1$

at $s = 4$

$2(4) - 6 = B(4-2)$

$2 = 2B$

$B = 1$

~~at s = 4~~

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

taking Inverse Laplace

$f(t) = e^{2t} + e^{4t}$

(vi) $\frac{5s-8}{s(s-4)}$

Solution

$$= \frac{A}{s} + \frac{B}{s-4}$$

taking limits

at $s=0$

$5(0)-8 = A(0-4)$

$-8 = -4A$

$A = 2$

at $s=4$

$5(4)-8 = B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

$$\frac{5s-8}{s(s-4)} \equiv \frac{2}{s} + \frac{3}{s-4}$$

taking inverse laplace

$f(t) = \underline{\underline{2u t + 3e^{4t}}} = \underline{\underline{2u(t) + 3e^{4t}}}$

(iv) $$\frac{s^3-3s-4}{(s-3)(s-1)^2}$$

solution

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} \equiv \frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)}$$

taking limits.

at $A=3$

$3^3 - 3(3)-4 = A(3-1)^2$

$27-9-4 = 4A$

$14 = 4A$

$A = 7/2$

for B

at $s=1$, $\frac{2s^3-9s^2+13}{(s-3)^2}$

$= \frac{2(1)^3-9(1)^2+13}{(1-3)^2} = \frac{2-9+13}{(-2)^2} = 6/4 = 3/2$

for c

at $s=1$ $= \frac{s^3-3s-4}{(s-3)} = \frac{(1)^3-3(1)-4}{(1-3)} = \frac{1-3-4}{-2} = \frac{-6}{-2} = 3$

$$f(t) = \mathcal{L}^{-1}\left[\frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{s-1}\right]$$

$$f(t) = \frac{7}{2}e^{3t} + 3/_2 e^{t} + 3e^{t}$$

v) $\frac{s-5}{s^2+4s+20}$

Solution

$$\frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)} = \frac{s-5}{(s+2-j4)(s+2+4j)}$$

$$A/s = -2+4j = \frac{(-2+4j)-5}{-2+4j+2+4j} = \frac{-7+4j}{8j} \times j/j$$

$$= \cancel{\frac{-7j-4}{-8}} = \frac{7j+4}{8}$$

$$B/s = -2-4j = \frac{-2-4j-5}{-2-4j+2-4j} = \frac{-7j+4}{8}$$

$$f(t) = 7j+4\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$$