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15/ENG02/033

Computer Engineering

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$W_1 = (1-x^2)y''$

$u = y''$ $\quad u^n = y^{(n+2)}$

$V = 1-x^2$ $\quad V' = -2x$ $\quad V'' = -2$ $\quad V''' = 0$

$W_1^n = y^{(n+2)}(1-x^2) + ny^{(n+1)}\cdot -2x + \frac{n(n-1)y^n \cdot -2}{2!} + 0$

$\cancel{W_1^n = y^{(n+2)}(1-x^2) + ny^{(n+1)}\cdot -2x + \frac{n(n-1)y^n \cdot -2}{2!} + 0}$

$W_1^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - (n^2+n)y^n$

$W_2 = -2xy'$

$V = -2x$ $\quad V' = -2$ $\quad V'' = 0$

$u = y'$ $\quad u^n = y^{(n+1)}$

$W_2^n = y^{(n+1)}\cdot -2x + ny^n \cdot -2 + 0$

$W_2^n = -2xy^{(n+1)} - 2ny^n$

$W_3 = 2y$

$V = 2$ $\quad V' = 0$

$u = y$ $\quad u^n = y^n$

$W_3^n = y^n \cdot 2 + 0$

$W_3^n = 2y^n$

$y^n = y^{(n+2)}(1-x^2) - 2xy^{(n+1)} - (n^2-n)y^n - 2xy^{(n+1)} - 2ny^n + 2y^n$

$y^n = (1-x^2)y^{(n+2)} - 2xy^{(n+1)} - 2xy^{(n+1)} - (n^2-n)y^n - 2ny^n + 2y^n$

$y^n = (1-x^2)y^{(n+2)} - (n+1)2xy^{(n+1)} - (n^2+n-2)y^n$

at $x = 0$

$(1-0^2)y^{(n+2)} - (n+1)2(0)y^{(n+1)} - (n^2+n-2)y^n$

$y^{(n+2)} - (n^2+n-2)y^n$

at $n = 0$

at $n=0$

$$(y^2)_0 = (0^2+0-2)(y^0)_0 = -2(y^0)_0$$

at $n=1$

$$(y^3)_0 = (1^2+1-2)[y^1]_0 = 0(y^1)_0$$

$n=2$

$$(y^4)_0 = [2^2+2-2][y^2]_0 = 4[y^2]_0 = -2\cdot 4[y^0]$$
$$= -8[y^0]_0$$

$n=3$ $[y^5] = [3^2+3-2][y^3]_0 = 10[y^3]_0 = 13\cdot 3[y^1]_0 = 3\cdot 0 = 0$

$n=4$ $[y^6] = (4^2+4-2][y^4] = 18[y^4]_0 = 18\cdot -8[y^0]_0 = 144[y^0]$

$n=5$ $[y^7] = [5^2+5-2][y^5] = 28[y^5]_0 = 28\cdot 3[y^1]_0 = 0$

$$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^4)_0 x$$

... ...

$$y = (y^0)_0 + x(y^1)_0 + x^2\cdot(y^0)_0 + \frac{x^3}{2}(y^1)_0 - \frac{x^4}{3}(y^0)_0 +$$

$$\frac{x^5}{5\times4\times3\times2}\cdot 39(y^1)_0 + \frac{x^6}{6\times5\times4\times3\times2}\cdot 144(y^0)_0 + \frac{x^7}{7!}\cdot 1209(y^1)_0$$

$$y = (y^0)_0 + x(y^1)_0 - x^2(y^0)_0 + \frac{x^3}{2}(y^1)_0 - \frac{x^4}{3}(y^0)_0 +$$

$$\frac{13x^5(y^1)_0}{40} + \frac{49x^6(y^0)_0}{240} + \frac{403x^7(y^1)_0}{1608}$$

$$y = (y^0)_0\left[1 - x^2 - \frac{x^4}{3} + \frac{3}{5}x^6 + \cdots\right] + (y^1)_0[x+0+0+0\cdots]$$

$$y = (y^0)_0\left[1 - x^2 - x^4/3 - \frac{3x^2}{5}\right] + x(y^1)_0$$

2) i $L[3e^{-4t} - 5e^{4t}]$

$L[3e^{-4t}] - L[5e^{4t}$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii) $L[\sin 4t + \cos 4t]$

$= L[\sin 4t] + L[\cos 4t]$

$= \dfrac{4}{s^2+16} + \dfrac{s}{s^2+16} = \dfrac{4+s}{s^2+16}$

viii) $t \sin 2t$

$L[\sin 2t] = \dfrac{2}{s^2+4}$

$L[t \sin 2t] = -1 \dfrac{d}{ds}\left[\dfrac{2}{s^2+4}\right]$

$= -1 \times \dfrac{4s}{(s^2+4)^2}$

ix) $t^3 + 4t^2 + 5$

$= L[t^3] + L[4t^2] + L[5]$

$= \dfrac{3!}{s^4} + \dfrac{4 \cdot 2!}{s^3} + \dfrac{5}{s}$

$x(s) = \dfrac{6}{s^4} + \dfrac{8}{s^3} + \dfrac{5}{s}$

vi) $t^2 \cos t$

$L[\cos t] = \dfrac{s}{s+1}$

$L[t^2 \cos t] = (-1)^2 \dfrac{d^2}{ds^2}\left[\dfrac{s}{s^2+1}\right] = 1 \times \dfrac{2s-6s^2}{(s^2+1)^3}$

$= \dfrac{2(s-3s^2)}{(s^2+1)^3}$

$= \dfrac{2(s-3s^3)}{(s^2+1)^3}$

x] $e^{3t}[t^2+4]$

$L[t^2+4] = \dfrac{2}{s^3} + \dfrac{4}{s-3}$

$$X(s) = L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$

//

vii) $L\left[\frac{\sinh 2t}{t}\right] = \frac{2}{s^2-4}$

$$X(s) = L\left[\frac{\sinh 2t}{t}\right] = \int_0^{\infty} \frac{2}{s^2-4} = 2\int_{s=\sigma}^{\infty} \frac{1}{\sigma^2-4}$$

$$2\left[\frac{1}{2}\tan^{-1}\frac{\sigma}{2}\right]_s^{\infty} \quad 2\left[\tan^{-1}\frac{\sigma}{2}\right]_s$$

$$= \tan^{-1}\frac{\infty}{2} - \tan^{-1}\frac{s}{2}$$

$$= \tan^{-1}\frac{2}{s}$$