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15/ENG02/082
ENG 381 Assignment

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\,dy/dx + 2y = 0$ — ①

This can be rewritten as

$(1-x^2)y'' - 2xy' + 2y = 0$ —— ②

Let $W_1 = (1-x^2)y''$

Solving using Leibnitz

$u = y''$ $\qquad V = 1-x^2$

$u^n = y^{n+2}$ $\qquad V' = -2x$

$V'' = -2$

$V''' = 0$

$W_1^n = y^{n+2}(1-x^2) + n\,y^{(n+2-1)}.(2x) + \frac{n(n-1)y^{(n+2-2)}.(-2)}{2!}$
$+ n(n-1)(n-2)\,y^{(n+2-3)}.0$

$W_1^n = (1-x^2)y(n+2) - 2xny^{(n+1)} - n(n-1)y^n$ — ③

Let $W_2 = -2xy'$

$u = y'$ $\qquad V = -2x$

$u^n = y^{(n+1)}$ $\qquad V' = -2$

$V'' = 0$

$W_2^n = y^{(n+1)}.(-2x) + ny^{(n+1-1)}.(-2) + n(n-1)y^{(n+1-2)}.0$

$= -2xy^{n+1} - 2ny^n$ —— ④

Let $W_3 = 2y$

$u = y$ $\qquad V = 2$

$u^n = y^n$ $\qquad V' = 0$

$W_3^n = 2y^n$

$y^n = W_1^n + W_2^n + W_3^n$

$y^n = (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} - 2x(n+1)y^{n+1} + (-n^2+n-2n+2)y^n$

$y^n = (1-x^2)y^{n+2} - 2x(n+1)y^{n+1} + (-n^2-n+2)y^n$

at $x=0$ and $y^n = 0$

$y^n = 0 = (1-0^2)y^{n+2} - 2(0)(n+1)y^{n+1} + (-n^2-n+2)y^n$

$= y^{n+2} + (-n^2-n+2)y^n$

$y^{n+2} = (n^2+n-2)y^n$

$n \geq 0$

$n = 0, 1, 2, 3, 4, 5 \ldots$

at $n=0$

$y^2 = -2(y)_0$

at $n=1$

$y^3 = (1+1-2)y^3$

$y^3 = 0$

at $n=2$

$y^4 = (2^2+2-2)y^2$

$y^4 = 4y^2 = 4[-2(y_0)] = -8[y_0]$

at $n=3$

$y^5 = (3^2+3-2)y^3$

$= (10y^3) = 10(y^3) = 0$

at $n=4$

$y^6 = (4^2+4-2)y^4$

$= (16+4-2)y^4 = 18(y^4)_0 = 18(-8y_0) = -144(y)_0$

at $n=5$

$y^7 = (5^2+5-2)y^5$

$= (28)y^5 = 28y^3y^5 = 0$

From maclaurins

$y(x) = y_0 + x(y')_0 + \frac{x^2}{2!}(-2(y)_0] + \frac{x^3}{3!}(0) + \frac{x^4}{4!}[-8[y_0]]$
$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}[-144(y)_0] + \frac{x^7}{7!}(0)$

$y(x) = y_0 + xy_0' - x^2y_0 - \frac{8}{24}x^4y_0 - \frac{144}{720}x^6y_0$

2) solve $3e^{-4t} - 5e^{4t} = f(t)$

i) $f(s) = L[3e^{-4t}) - L[5e^{4t}]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

$= \frac{-2s-32}{s^2-16}$

ii) $\sin 4t + \cos 4t = f(t)$

$f(s) = L[\sin 4t] + L[\cos 4t]$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$

$= \frac{s^3+4s^2+16s+64}{(s^2+16)^2}$

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iii) $t^3 + 2t^2 - t + 4 = f(t)$

$F(s) = L[t^3] + L[2t^2] - L[t] + L[4]$

$= \frac{3!}{s^4} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s} + 4/s$

$= \frac{4s^3 - s^2 + 4s + 6}{s^4}$

iv) $e^{-2t}\cos 5t = f(t)$

$f(s) = L[e^{-2t}\cos 5t]$

$= \frac{s+5}{(s+5)^2 + 5^2}$

$= \frac{s+5}{s^2 + 10s + 50}$

v) $t \sin 3t = f(t)$

$f(s) = L[t \sin 3t]$

$= \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$

Using quotient rule

$f(s) = -1\left[\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2}\right]$

$= \frac{6s}{(s^2+9)^2}$

vi) $\frac{e^{-t} - e^{-2t}}{t} = f(t)$

$f(s) = L\left[\frac{1}{t}[e^{-t} - e^{-2t}]\right]$

$= \int \left(\frac{1}{s+1} - \frac{1}{s+2}\right)$

$= \int \frac{1}{s^2+3s+2} ds$

$f(s) = \ln(s^2+3s+2)$

vii) $e^{4t}\cos 2t = f(t)$

$f(s) = L(e^{4t}\cos 2t]$

$= \frac{s-4}{(s-4)^2+4^2} = \frac{s-4}{s^2-8s+32}$

viii) $t \sin 2t = f(t)$

$f(s) = L[t \sin 2t]$

$= -\frac{d}{ds}\left[\frac{2}{s^2+4}\right]$

(Using quotient rule) $f(s) = -\left[\frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2}\right]$

$= \frac{4s}{(s^2+4)^2}$

ix) $t^3+4t^2+5 = f(t)$

$f(s) = L[t^3+4t^2+5]$

$= 3!/s^4 + 4\left[2!/s^3\right] + 5/s$

$= 6/s^4 + 8/s^3 + 5/s$

x) $e^{3t}(t^2+4) = f(t)$

$e^{3t}t^2 + 4e^{3t} = f(t)$

$f(s) = L[e^{3t}t^2] + L[4e^{3t}]$

$f(s) = \frac{2!}{(s-3)^3} + \frac{4}{s-3}$

$= \frac{4s^2-26s+30}{(s-3)^3}$

3) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-4} + \frac{B}{s-3}$

at $s=4$

$4-5 = A(4-3)$

$-1 = A \quad \therefore A = -1$

at $s=3$

$3-5 = B(3-4)$

$-2 = -B \quad \therefore B = 2$

$\frac{s-5}{(s-3)(s-4)} = \frac{-1}{s-4} + \frac{2}{s-3}$

$f(t) = -e^{4t} + 2e^{3t}$

$= 2e^{3t} - e^{4t}$

ii) $\dfrac{2s-6}{(s-2)(s-4)}$

Solution

$\dfrac{A}{s-2} + \dfrac{B}{s-4}$

at $s = 2$

$2(2) - 6 = A(2-4)$

$-2 = -2A \quad \therefore A = 1$

at $s = 4$

$2(4) - 6 = B(4-2)$

$2 = 2B \quad \therefore B = 1$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$f(t) = e^{2t} + e^{4t}$

iii) $\dfrac{5s-8}{s(s-4)}$

Solution

$\dfrac{A}{s} + \dfrac{B}{s-4}$

taking limits at $s = 0$

$5(0) - 8 = A(0-4)$

$4A = 8 \quad \therefore A = 2$

at $s = 4$

$5(4) - 8 = B(4)$

$20 - 8 = 4B$

$12 = 4B$

$B = 3$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

taking inverse laplace

$f(t) = 2u(t) + 3e^{4t}$

iv) $$\frac{s^3 - 3s - 4}{(s-3)(s-1)^2}$$

Solution

$$\frac{s^3 - 3s - 4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

at $A = 3$

$3^3 - 3(3) - 4 = A(3-1)^2$

$27 - 9 - 4 = 4A$

$14 = 4A \quad \therefore A = 7/2$

at $s=1$ for B $\quad \frac{2s^3 - 9s^2 + 13}{(s-3)^2}$

$$\frac{2(1)^3 - 9(1)^2 + 13}{(1-3)^2} = \frac{2 - 9 + 13}{-2^2} = 3/2$$

for C

at $s=1$ $= \frac{s^3-3s-4}{s-3} = \frac{(1)^3-3(1)-4}{(1-3)} = \frac{1-3-4}{-2}$

$= \frac{-6}{-2} = 3$

$f(t) = L^{-1}\left[\frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{(s-1)}\right]$

$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}e^{t} + 3e^{t}$

v) $= \frac{A}{(s+2-j4)} + \frac{B}{s+2+4j} = \frac{s-5}{(s+2-j4)(s+2+4j)}$

$A/s = -2+4j = \frac{(-2+4j)-5}{-2+4j+2+4j} = \frac{-7+4j}{8j} \times \frac{j}{j}$

$= \frac{7j+4}{8}$

$B/s = -2-4j = \frac{-2-4j-5}{-2-4j+2-4j} = \frac{-7j+4}{8}$

$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$