ASSIGNMENT 4: Gwaze Daniel Danladi

15/ENG03/013

CIVIL ENG

300Level

① $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$ — (1)

eqn1) can also be written as

$(1-x^2)y'' - 2xy' + 2y = 0$ —— (2)

taking the first term $(1-x^2)y''$

$u = y''$ $\quad v = 1-x^2$

$u^n = y^{(n+2)}$ $\quad v' = -2x$

$v'' = -2$

$v''' = 0$

$y^n = y^{(n+2)}(1-x^2) + ny^{(n+2-1)}\cdot -2x + n(n-1)y^{(n+2-2)}\cdot -2 + n(n-1)(n-2)y^{(n+2-3)}\cdot 0$

$y^n = (1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)}$ — (3)

taking the second term $-2xy'$

$u = y'$ $\quad V = -2x$

$u^n = y^{(n+1)}$ $\quad V' = -2$

$V'' = 0$

$y^n = y^{(n+1)}\cdot -2x + ny^{(n+1-1)}\cdot -2 + n(n-1)y^{(n+1-2)}\cdot 0$

$y^n = -2xy^{(n+1)} - 2ny^{(n)} + 0$ —— (4)

taking the third term $2y$

$u = y$ $\quad V = 2$

$u^n = y^n$ $\quad V' = 0$

$y^n = y^n 2 + ny^{(n-1)}\cdot 0$

$y^n = 2y^n$ —— (5)

∴ taking the three equations (3, 4 and 5) we have

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$$(1-x^2)y^{(n+2)} - 2xny^{(n+1)} - 2n(n-1)y^{(n)} - 2xy^{(n+1)}$$
$$-2ny^{(n)} + 2y^{(n)} = 0$$

$$(1-x^2)y^{(n+2)} - 2x(ny^{(n+1)} + y^{(n+1)}) + y^{(n)}(-2n+2) = 0 \quad \text{———} \quad (6)$$

Now at $x=0$, eqn 6 becomes

$$y^{(n+2)} + 2y^{(n)}(-n+1) = 0$$

$$y^{(n+2)} = -(-n+1)2y^{(n)}$$
$$y^{(n+2)} = (n-1)2y^{(n)} \quad \text{———} \quad (7)$$

eqn(7) gives the recurrence equation.

now at $n = 0$

$$y^2 = (0-1)2y^{(0)}$$
$$(y^2)_0 = -2y_0 \quad \text{———} \quad (7)$$

now at $n = 1$

$$(y^3)_0 = 0 \quad \text{———} \quad (8)$$

now at $n = 2$

$$(y^4)_0 = 2(y^2)_0 = 2(-2y_0) = -4y_0 \quad \text{———} \quad (9)$$

now at $n = 3$

$$(y^5)_0 = 4(y^{(3)})_0 = 4(0) = 0 \quad \text{—} \quad (10)$$

now at $n = 4$

$$(y^6)_0 = 6y^{(4)} = 6(-4y_0) = -24y_0 \quad \text{—} \quad (11)$$

now at $n = 5$

$$(y^7)_0 = 0 \quad \text{———} \quad (12)$$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

now at $n = 6$

$$(y^8)_0 = 10(y^{(6)})_0 = 10(-24\, y_0)$$
$$= -240\, y_0 \quad \text{— (13)}$$

Recall the maclaurin method

$$y(x) = y_0(x) + x y'_0(x) + \frac{x^2}{2!} y''_0(x) + \frac{x^3}{3!} y'''_0(x) + \frac{x^4}{4!} y^4_0(x) + \frac{x^5}{5!} y^5_0(x) + \frac{x^6}{6!} y^6_0(x) + \dots \frac{x^n}{n!} y^n_0(x) \quad \text{— (14)}$$

now substituting eqn (4, 8, 9, 10, 11, 12 & 13) into eqn (14) we have;

$$y(x) = y_0(x) + x y'_0 + \frac{x^2}{2!}(-2 y_0) + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(-4 y_0) + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(-24 y_0) + \frac{x^7}{7!}(0) + \frac{x^8}{8!}(-240 y_0)$$

$$y(x) = y_0\left(-2\frac{x^2}{2!} - \frac{x^4\,4}{4!} - \frac{24}{6!}x^6 - \frac{240}{8!}x^8\right) + y'_0 x + 0$$

$$y(x) = y_0\left(-x^2 - \frac{x^4}{6} - \frac{x^6}{30} - \frac{x^8}{168}\right) + y'_0 x$$

ASSIGNMENT 4: Gwaze Daniel Danladi

15/ENG03/013

CIVIL ENG

300Level

(2) (i) $3e^{-4t} - 5e^{4t} = f(t)$

- this is a linear equation ∴ the laplace transform

$L[f(t)] = f(s)$

$f(s) = L[3e^{-4t}] - L[5e^{4t}]$

$f(s) = 3\frac{1}{s+4} - 5\frac{1}{s-4}$ //

(ii) $\sin 4t + \cos 4t = f(t)$

$f(s) = L[f(t)]$

$f(s) = L[\sin 4t] + L[\cos 4t]$

$f(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

(iii) $t^2 + 2t^2 - t + 4 = f(t)$

$L[f(t)] = f(s)$

$f(s) = L(t^2] + 2L[t^2] - L[t] + L[4]$

$f(s) = \frac{2!}{s^3} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$f(s) = \frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t} \cos 5t$

$f(s) = L(e^{-2t} \cos 5t)$

first taking the laplace transform of $\cos 5t$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$L(\cos 5t) = \frac{s}{s^2+25}$

$\therefore$

$F(s) = \frac{s+2}{(s+2)^2+25}$

(v) $t \sin 2t$

taking the laplace transform of $\sin 2t$

$= \frac{3}{s^2+9}$

now taking the first formatting according to the power of t $(n = 1)$

recall $= -\frac{d}{ds}\frac{3}{s^2+9}$

Using product rule

$-1\left(\frac{s^2+9(0) - 3(2s)}{(s^2+9)^2}\right)$

$f(s) = -1\left[\frac{-6s}{(s^2+9)^2}\right]$

$f(s) = \frac{6s}{(s^2+9)^2}$ //

(vi) $\frac{e^{-t} - e^{-2t}}{t} \Rightarrow \frac{1}{t}\left[e^{-t} - e^{-2t}\right]$

$f(t) = \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$

$f(t) = t^{-1}e^{-t} - t^{-1}e^{-2t}$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$$f(s) = \frac{-1!}{(s+1)^{-1+1}} + \frac{1!}{(s+2)^{-1+2}}$$

$$f(s) = \frac{-1}{1} + \frac{1}{(s+2)}$$

$$f(s) = -1 + \frac{1}{(s+2)}$$

$$-s-2-1$$

$$\frac{-s-s}{(s+2)}$$

$$f(s) = \frac{1-s-2}{(s+2)} //$$

vii) $e^{4t} \cos 2t$

taking the laplace transform of $\cos 2t$

$$L(\cos 2t) = \frac{s}{s^2+4}$$

$\therefore$ the $L[e^{4t} \cos 2t] = \dfrac{s-4}{(s-4)^2+4} //$

viii) $t \sin 2t$

taking the laplace transform of $\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

now taking the derivative of the transform according to the power of t $(n=1)$

$$-\frac{d}{ds}\frac{2}{s^2+4}$$

$$= -\left[\frac{s^2+4(0) - 2(2s)}{(s^2+4)^2}\right]$$

$$= \frac{4s}{(s^2+4)^2} //$$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

(1 x) $t^3 + 4t^2 + 5 = f(t)$

$f(s) = L[f(t)]$

$f(s) = L[t^3] + L[4t^2] + L[5]$

$f(s) = \frac{3!}{s^4} + 4\frac{2!}{s^3} + 5/s$

$f(s) = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $t^2 \cos t$

taking the laplace trans form of cos t

$L[\cos t] = \frac{s}{s^2+1}$

now taking the derivative of the trans form according to the power of t (n=2)

$f(s) = -\frac{d}{ds}\left(-\frac{d}{ds}\frac{s}{s^2+1}\right)$

$= -\left[\frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}\right]$

$= -\left[\frac{s^2+1-4s}{(s^2+1)^2}\right]$

$= \frac{-1-s^2+4s}{(s^2+1)^2} = \frac{4s-s^2-1}{(s^2+1)^2}$

now taking the second derivative

$-\left[\frac{(s^2+1)^2(4-2s) - (4s-s^2-1)(4s^3+4s)}{(s^2+1)^4}\right]$

$= -\left[\frac{(s^4+2s^2+1)(4-2s) - (4s-s^2-1)(4s^2+4s)}{(s^2+1)^4}\right]$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$$-\left[\frac{(s^4+2s^2+1](4-2s) - (16s^3+16s^2-4s^5-4s^3-4s^3-4s)}{(s^2+1)^4}\right]$$

$$-\left[\frac{(s^4+2s^2+1)(4-2s) - (-4s^5+8s^3+16s^2-4s)}{(s^2+1)^4}\right]$$

$$-\left[\frac{4s^4+2s^5+8s^2-4s^3+4-2s+4s^5-8s^3-16s^2+4s}{(s^2+1)^4}\right]$$

$$-\left[\frac{4s^4+2s^5-8s^2-12s^3+2s+4}{(s^2+1)^4}\right]$$

$$= \frac{-4s^4-2s^5+8s^2+12s^3-2s-4}{(s^2+1)^4}$$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

No 2

i $3e^{-4t} - 5e^{4t} = f(t)$

$L[f(t)] = f(s)$

$f(s) = L[3e^{-4t}] - L[5e^{4t}]$

$f(s) = 3\frac{1}{s+4} - 5\frac{1}{s-4}$

$f(s) = \frac{3}{s+4} - \frac{5}{s-4}$ //

ii $\sin 4t + \cos 4t = f(t)$

$f(s) = L[f(t)]$

$f(s) = L[\sin 4t] + L[\cos 4t]$

$f(s) = \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

iii $t^2 + 2t^2 - t + 4 = f(t)$

$L[f(t)] = f(s)$

$f(s) = \frac{2!}{s^3} + 2\left[\frac{2!}{s^3}\right] - \frac{1}{s^2} + \frac{4}{s}$

$f(s) = \frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$ //

iv $e^{-2t} \cos 5t = f(t)$

$f(s) = L[f(t)]$

$L[\cos 5t] = \frac{s}{s^2+25}$

$f(s) = \frac{s+2}{(s+2)^2 + 25}$ //

v.) $t \sin 3t = f(t)$

$L[\sin 3t] = \frac{3}{s^2+9}$

recall $f(s) = -\frac{d^n}{ds^n} \frac{3}{s^2+9}$

Using product rule

$f(s) = -1\left[\frac{s^2+9(0) - 3(2s)}{(s^2+9)^2}\right]$

$f(s) = -1\left[\frac{-6s}{(s^2+9)^2}\right]$

$f(s) = \frac{6s}{(s^2+9)^2}$ //

vi). $\frac{e^{-t} - e^{-2t}}{t} = f(t)$

$f(t) = t^{-1}e^{-t} - t^{-1}e^{-2t}$

$f(s) = \frac{-1!}{(s+1)^{-1+1}} + \frac{1!}{(s+2)^{-1+2}}$

$f(s) = \frac{-1}{1} + \frac{1}{s+2}$

$f(s) = -1 + \frac{1}{s+2}$

$f(s) = \frac{-s-3}{(s+2)}$ //

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

vii) $e^{4t}\cos 2t = f(t)$

$$L[\cos 2t] = \frac{s}{s^2+4}$$

$$f(s) = \frac{s-4}{(s-4)^2+4}$$ //

viii) $t\sin 2t$

$$L[\sin 2t] = \frac{2}{s^2+4}$$

Recall

$$f(s) = -\frac{d^n}{ds^n}\frac{2}{s^2+4}$$

$$f(s) = -\left[\frac{s^2+4(0)-2(2s)}{(s^2+4)^2}\right]$$

$$f(s) = \frac{4s}{(s^2+4)^2}$$ //

ix $t^3+4t^2+5 = f(t)$

$$f(s) = L(f(t))$$

$$f(s) = \frac{3!}{s^4} + 4\frac{2!}{s^3} + 5/s$$

$$f(s) = 6/s^4 + 8/s^3 + 5/s$$ //

x) $e^{3t}(t^2+4) = f(t)$

$$\Rightarrow t^2e^{3t} + 4e^{3t}$$

$$f(s) = L(f(t))$$

$$f(s) = \frac{2}{(s-3)^3} + \frac{4}{s-3}$$ //

(xi) $t^2\cos t$

$$f(s) = (-1)^n\frac{d^n}{ds^n}$$

$$L[\cos t] = \frac{s}{s^2+1}$$

$$f(s) = (-1)^2\frac{d^2 s}{ds^2(s^2+1)}$$

taking the first derivative using quotient rule

$$-1\left[\frac{s^2+1(1)-s(2s)}{(s^2+1)^2}\right]$$

$$\Rightarrow -\left[\frac{s^2+1-2s^2}{(s^2+1)^2}\right] = \frac{-1+s^2}{(s^2+1)^2}$$

taking the second derivative

$$\Rightarrow \frac{(s^2+1)^2(-2s)-(1-s^2)(4s^3+4s)}{(s^2+1)^4}$$

$$\Rightarrow \frac{(s^2+1)^2(+2s)+(s^2+1)(4s^3+4s)}{(s^2+1)^4}$$

$$\Rightarrow \frac{\cancel{(s^2+1)}[2s-4s^3+4s]}{\cancel{(s^2+1)}^4}$$

ASSIGNMENT 4: Gwaze Daniel Danladi

15/ENG03/013

CIVIL ENG

300Level

$$f(s) = \frac{6s - 4s^3}{(s^2+1)^3} \quad //$$

(x ii) $\frac{\sinh 2t}{t} = f(t)$

$f(t) = t^{-1} \sinh 2t$

$L[\sinh 2t] = \frac{a}{s^2 - a^2}$

$f(s) =$

$\frac{2}{s^2+4} + 3$

$\frac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$

$14 + 3s^2$

③

i $\frac{s-5}{(s-3)(s-4)} = f(s)$

$\frac{A}{(s-4)} + \frac{B}{(s-3)} = \frac{s-5}{(s-4)(s-3)}$

$A|_{s=4} = \frac{4-5}{(4-3)} = \frac{-1}{1} = -1$

$B|_{s=3} = \frac{3-5}{(3-4)} = \frac{-2}{-1} = 2$

$A = -1$ and $B = 2$

$f(s) = \frac{-1}{(s-4)} + \frac{2}{(s-3)}$

$f(t) = L^{-1}[f(s)]$

$f(t) = -e^{4t} + 2e^{3t}$

$f(t) = 2e^{3t} - e^{4t} \quad //$

ii $\frac{2s-6}{(s-2)(s-4)}$

$\frac{A}{(s-2)} + \frac{B}{(s-4)} = \frac{2s-6}{(s-2)(s-4)}$

$A|_{s=2} = \frac{2(2)-6}{(2-4)} = 1$

ASSIGNMENT 4: Gwaze Daniel Danladi

15/ENG03/013

CIVIL ENG

300Level

$$B|_{s=4} = \frac{2(4)-6}{(4-2)} = 1$$

$A = 1$ and $B = 1$

$$f(s) = \frac{1}{s-2} + \frac{1}{s-4}$$

$$f(t) = L^{-1}[f(s)]$$

$$f(t) = e^{2t} + e^{4t}$$ //

iii $$\frac{5s-8}{s(s-4)} = f(s)$$

$$= \frac{A}{s} + \frac{B}{s-4} = \frac{5s-8}{s(s-4)}$$

$$A|_{s=0} = \frac{5(0)-8}{(0-4)} = 2$$

$$B|_{s=4} = \frac{5(4)-8}{4} = 3$$

$A = 2$ and $B = 3$

$$f(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$f(t) = L^{-1}[f(s)]$$

$$f(t) = 2 + 3e^{4t}$$ //

iv) $$\frac{s^3-3s-4}{(s-3)(s-1)^2} = f(s)$$

$$\frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1} = \frac{s^3+3s-4}{(s-3)(s-1)^2}$$

$$A|_{s=3} = \frac{(3)^3-3(3)-4}{(3-1)^2}$$

$$= \frac{27-9-4}{4} = 14/4$$

$$= 7/2$$

$$B|_{s=1} = \frac{d}{ds}\frac{s^3-3s-4}{s-3}$$

$$B|_{s=1} = \frac{2s^3-9s^2+13}{(s-3)^2}$$

$$B|_{s=1} = \frac{2-9+13}{4} = \frac{6}{4}$$

$$B = 3/2$$

$$C|_{s=1} = \frac{s^3-3s-4}{(s-3)}$$

$$= \frac{(1)^3-3(1)-4}{(1-3)} = 3$$

$A = 7/2$, $B = 3/2$, $C = 3$

$$f(s) = \frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{(s-1)}$$

$$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}te^{t} + 3e^{t}$$ //

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

③ Convert the following functio[illegible] time (t) domains!

(i) $\frac{s-5}{(s-3)(s-4)}$

$$\frac{A}{(s-4)} + \frac{B}{(s-3)} = \frac{s-5}{(s-3)(s-4)}$$

$$A(s-3) + B(s-4) = s-5$$

$$A|s=4 \Rightarrow \frac{4-5}{(4-3)} = \frac{-1}{1} = -1$$

$$B|s=3 \Rightarrow \frac{3-5}{(3-4)} = \frac{-2}{-1} = 2$$

$A = -1$ and $B = 2$

$$f(s) = \frac{-1}{(s-4)} + \frac{2}{(s-3)}$$

$$f(t) = L^{-1}[f(s)]$$

$$f(t) = -1e^{4t} + 2e^{3t}$$ //

(ii) $\frac{2s-6}{(s-2)(s-4)}$

$$\frac{A}{(s-2)} + \frac{B}{(s-4)} = \frac{2s-6}{(s-2)(s-4)}$$

$$A|s=2 = \frac{2(2)-6}{(2-4)} = \frac{-2}{-2} = 1$$

$$B|s=4 = \frac{2(4)-6}{(4-2)} = \frac{2}{2} = 1$$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$A = 1$ and $B = 1$

$$f(s) = \frac{1}{(s-2)} + \frac{1}{(s-4)}$$

$$f(t) = L^{-1}\left[f(s)\right]$$

$$f(t) = e^{2t} + e^{4t} \quad //$$

iii $\frac{5s-8}{s(s-4)}$

$$\frac{A}{s} + \frac{B}{s-4} = \frac{5s-8}{s(s-4)}$$

$$A|_{s=0} = \frac{5(0)-8}{(0-4)} = \frac{-8}{-4} = 2$$

$$B|_{s=4} = \frac{5(4)-8}{4} = \frac{12}{4} = 3$$

$A = 2$ and $B = 3$

$$f(s) = \frac{2}{s} + \frac{3}{s-4}$$

$$f(t) = L^{-1}\left[f(s)\right]$$

$$f(t) = 2u(t) + 3e^{4t} \quad //$$

(iv) $\frac{s^3-3s-4}{(s-3)(s-1)^2}$

$$\frac{A}{(s-3)} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)} = \frac{s^3-3s-4}{(s-3)(s-1)^2}$$

$$A|_{s=3} = \frac{(3)^3-3(3)-4}{(3-1)^2} = \frac{27-9-4}{4} = \frac{14}{4}$$

$$= \frac{7}{2}$$

ASSIGNMENT 4: Gwaze Daniel Danladi
15/ENG03/013
CIVIL ENG
300Level

$$B|_{s=1} = \frac{d}{ds}\frac{s^3-3s-4}{(s-3)}$$

$$\frac{(s-3)(3s^2-3)-(s^3-3s-4)(1)}{(s-3)^2}$$

$$B|_{s=1} = \frac{3s^3-\cancel{3s}-9s^2+9-s^3+\cancel{3s}+4}{(s-3)^2}$$

$$B|_{s=1} = \frac{2s^3-9s^2+13}{(s-3)^2}$$

$$= \frac{2(1)^3-9(1)^2+13}{(1-3)^2} = \frac{2-9+13}{(-2)^2} = \frac{6}{4} = \frac{3}{2}$$

$$C|_{s=1} = \frac{s^3-3s-4}{(s-3)} = \frac{(1)^3-3(1)-4}{(1-3)} = \frac{1-3-4}{-2}$$

$$C|_{s=1} = \frac{-6}{-2} = 3$$

$$f(s) = \frac{7}{2(s-3)} + \frac{3}{2(s-1)^2} + \frac{3}{(s-1)}$$

$$f(t) = \frac{7}{2}e^{3t} + \frac{3}{2}e^{t} + 3e^{t} \; //$$

(v) $$\frac{s-5}{s^2+4s+20}$$

$$\frac{A}{(s+2-j4)} + \frac{B}{(s+2+4j)} = \frac{s-5}{(s+2-j4)(s+2+4j)}$$

$$A|_{s=-2+4j} = \frac{(-2+4j)-5}{(-\cancel{2}+4j+\cancel{2}+4j)} = \frac{-7+4j}{8j} \times \frac{j}{j}$$

$$A|_{s=-2+4j} = \frac{-7j-4}{-8} = \frac{7j+4}{8}$$

ASSIGNMENT 4: Gwaze Daniel Danladi

15/ENG03/013

CIVIL ENG

300Level

$$B|_{s=-2-4j} = \frac{-2-4j-5}{(-\cancel{2}-4j+\cancel{2}-4j)} = \frac{-7-4j}{-8j} \times \frac{j}{j}$$

$$B|_{s=-2-4j} = \frac{-7j+4}{8}$$

$$f(s) = \frac{7j+4}{8(s+2-4j)} - \frac{7j+4}{8(s+2+4j)}$$

$$f(t) = L^{-1}[f(s)]$$

$$f(t) = \frac{7j+4}{8} e^{(-2+4j)t} - \frac{7j+4}{8} e^{(-2-4j)t}$$

$$f(t) = \frac{7j+4}{8}\left(e^{(-2+4j)t} - e^{(-2-4j)t}\right)$$