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DEPT: MECH ENG

COURSE: ENG 381

Assignment IV

1) $(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' + 2xy' + 2y = 0$

For Leibnitz theorem, $\sum_{r=0}^{n} {}^nC_r U^{n-r} V^{r}$

for sub 1 $\quad V = (1-x^2) \qquad U = y''$

$\frac{dv}{dx} = -2x \qquad U^n = y^{n+2}$

$\frac{d^2v}{dx^2} = -2 \qquad U^{n-1} = y^{n+1} \qquad U^{n-2} = y^n$

$\therefore y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2}y^n(-2)$

for sub 2 : $\quad V = (2x) \qquad U = y' \quad U^n = y^{n+1}$

$\frac{dv}{dx} = 2 \qquad U^{n-1} = y^n$

$\therefore y^{n+1}(2x) + ny^n(2)$

for sub 3: $\quad V = 0 \qquad U = 2y$

$U^n = 2y^n$

$\therefore (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n + 2xy^{n+1} + 2ny^n + 2y^n = 0$

setting $x = 0$

$y_0^{n+2} - n(n-1)y_0^n + 2ny_0^n + 2y_0^n = 0$

$y_0^{n+2} = n(n-1)y_0^n - 2ny_0^n - 2y_0^n$

when $n = 0$: $(y_0^2) = -2y_0^0$

$n = 1$: $(y_0^3) = -2y_0^1 - 2y_0^1 = -4y_0^1$

$n = 2$ $y_0^4 = 2y_0^2 - 4y_0^2 - 2y_0^2 = -4(y_0^2) = -4(-2y_0^0) = 8y_0^0$

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if $n=3$ $\quad y_0^5 = 6y_0^3 - 6y_0^3 - 12y_0^3$

$= -2y_0^3 = -2(-4(y')_0) = 8y'_0)$

if $n=4$, $\quad y_0^6 = 12y_0^4 - 8y_0^4 - 2y_0^4 = -2y_0^4$

$= 2(8y_0^0) = 16y_0^0$

if $n=5$, $\quad y_0^7 = 20(y_0^5) - 10(y_0^5) - 2y_0^5$

$= 8(y_0^5) = 8(8(y'_0)) = 64y'_0$

Using maclaurin series : $\quad y = y_0 + xy'_0 + \frac{x^2}{2!}y''_0 + \frac{x^3}{3!}y'''_0 + \frac{x^n y^n}{n!}$

$\therefore y = y_0 + xy'_0 + \frac{x^2}{2!}(-2y_0) + \frac{x^3}{3!}(-4y'_0) = \frac{x^4}{4!}(8y_0^0) + \frac{x^5}{5!}(8y'_0)$

$+ \frac{x^6}{6!}(16y_0^0) + \frac{x^7}{7!}(64y'_0)$

~~$y = \frac{4t}{} y_0$~~ $\quad y = y_0 + xy'_0 - x^2y_0 - \frac{2x^3y'_0}{3} + \frac{x^4y_0^0}{3} + \frac{x^5y'_0}{15} + \frac{x^6y_0^0}{45} + \frac{4x^7y'_0}{315}$

$\therefore y = y_0\left[1 - x^2 + \frac{x^4}{3} + \frac{x^6}{45}\right] + y'_0\left[x - \frac{2x^3}{3} + \frac{x^5}{15} + \frac{4}{315}x^7\right]$

(2) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\} \Rightarrow L\{3e^{-4t}\} - L\{5e^{4t}\}$

$\Rightarrow \frac{3}{s+4} - \frac{5}{s-4}$

(3) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + \cos 4t\}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$

(4) $t^3 - 2t^2 - t + 4 \quad = L\{t^3\} - L\{2t^2\} - L\{t\} + L\{4\}$

$= \frac{6}{s^4} - \frac{4}{s^3} - \frac{2}{s^2} + \frac{4}{s}$

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iv) $e^{-2t}\cos 5t \Rightarrow L\{e^{2t}\} \cdot L\{\cos 5t\} = L\{e^{-2t}\cos 5t\}$

$L\{\cos 5t\} \Rightarrow \dfrac{s}{s^2+25} \quad \cdot \quad \dfrac{s+2}{(s+2)^2+25}$

v) $t\sin 3t \qquad L\{\sin 3t\}$

$L\{\sin 3t\} = \dfrac{3}{s^2+9} \qquad F'(s) = \dfrac{-d}{ds}\left[\dfrac{3}{s^2+9}\right]$

$\dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \dfrac{(s^2+9)(0) - (3)(2s)}{(s^2+9)^2} = \dfrac{-6s}{(s^2+9)^2}$

(vi) $e^{-t} - e^{-2t} = L\left\{\dfrac{e^{-t}}{t} - \dfrac{e^{-2t}}{t}\right\}$

$L\left\{\dfrac{e^{t}}{t}\right\} - L\left\{\dfrac{e^{-2t}}{t}\right\} = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$\int_{\sigma=s}^{\infty} \dfrac{1}{\sigma+1}\,d\sigma - \int_{\sigma=s}^{\infty} \dfrac{1}{\sigma+2}\,d\sigma$

$[\ln(\sigma+1)]_s^{\infty} - \ln(\sigma+2)]_s^{\infty} = -\ln(s+1) + \ln(s+2)$

$= +\ln(s+2) - \ln(s-1 \Rightarrow \ln\left(\dfrac{s+2}{s-1}\right)$

vii) $e^{4t}\cos 2t \qquad L\{e^{4t}\cos 2t\}$

$L\{\cos 2t\} = \dfrac{s}{s^2+2^2} = \dfrac{s}{s^2+4}$

$L\{e^{4t}\cos 2t\} = \dfrac{(s-4)}{(s-4)^2+4}$

viii) $t\sin 2t = L\{t\sin 2t\}$

$L\{\sin 2t\} = \dfrac{2}{s^2+4} \qquad L\{t\sin 2t\} = \dfrac{d}{ds}\left[\dfrac{2}{s^2+4}\right]$

$\dfrac{u\frac{dv}{dx} - u\frac{dv}{dx}}{v^2} = \dfrac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2} = \dfrac{-4s}{(s^2+4)^2}$

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1p) $t^3+4t^2+5$

$L\{t^3+4t^2+5\} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

p) $e^{3t}(t^2+4)$

$L\{t^2+4\} = \frac{2}{s^3} + \frac{4}{s}$

$L\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^3} + \frac{4}{(s-3)} = \frac{2+4(s-3)^2}{(s-3)^3}$

v) $t^2\cos t$ $= L\{t^2\cos t$

$L\{\cos t\} = \frac{s}{s^2+2}$ $\qquad L\{t\cos t\} = \frac{d}{ds}\left(\frac{s}{s^2+2}\right)$

$\frac{u\frac{dv}{dx} - u\frac{dv}{ds}}{v^2} = \frac{s^2+2(1) - (s)(2s)}{(s^2+2)^2} = \frac{s^2+2-2s^2}{(s^2+2)^2} = \frac{2-s^2}{(s^2+2)^2}$

$L\{t^2\cos t\} = \frac{d}{ds}\left(\frac{2-s^2}{(s^2+2)^2}\right) = \frac{(s^2+2)^2(-2s) - (2-s^2)(4s(s^2+2))}{(s^2+2)^4}$

$= \frac{(s^2+2)\left((s^2+2)(-2s) - (2-s^2)(4s)\right)}{(s^2+2)^4}$

$= \frac{(s^2+2)[-2s^3-4s - 8s + 4s^3]}{(s^2+2)^4} = \frac{2s^3-12s}{(s^2+2)^3}$

vii) $\frac{\sinh 2t}{t}$ $\qquad L\{\sinh 2t\} = \frac{2}{s^2-4}$

$F_\sigma = \frac{2}{s^2-4}$ $\qquad \int_{\sigma=s}^{\infty} \frac{2}{s^2-4}\, d\sigma = \left[-\tan^{-1}\frac{\sigma}{2}\right]_s^{\infty}$

$= \left[\tan^{-1}\frac{s}{2} - \tan^{-1}\frac{\infty}{2}\right] = \tan^{-1}\frac{s}{2} - 90 = -\tan^{-1}\frac{2}{s}$

Ruitai paper

3(i) $\dfrac{s-5}{(s-3)(s-4)} = \dfrac{A}{s-3} + \dfrac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

at $s=4$ : $4-5 = B(4-3)$ $B = -1.$

at $s=3$ : $3-5 = A(3-4)$ $A = 2.$

$\therefore L^{-1}\left\{\dfrac{s-5}{(s-3)(s-4)}\right\} = L^{-1}\left[\dfrac{2}{s-3} - \dfrac{1}{s-4}\right]$

$L^{-1}\left\{\dfrac{s-5}{(s-3)(s-4)}\right\} = 2e^{3t} - e^{4t}.$

3(ii) $\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{s-2} + \dfrac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

at $s=4$ $8-6 = 2B$ $B = 1.$

at $s=2$ $4-6 = -2A$ $A = 1$

$L^{-1}\left\{\dfrac{2s-6}{(s-2)(s-4)}\right\} = \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$\therefore L\left\{\dfrac{2s-6}{(s-2)(s-4)}\right\} = e^{2t} + e^{4t}$

3(iii) $\dfrac{5s-8}{s(s-4)} = \dfrac{A}{s} + \dfrac{B}{s-4}$

$5s-8 = A(s-4) + Bs$

At $s=4$ : $12 = 4B$ $B = 3.$

at $s=0$ : $-8 = -4A$ $A = 2$

$\therefore L^{-1}\left\{\dfrac{5s-8}{s(s-4)}\right\} = L^{-1}\left\{\dfrac{2}{s} + \dfrac{3}{s-4}\right\}$

$L^{-1}\left\{\dfrac{5s-8}{s(s-4)}\right\} = 2 + 3e^{4t}$

(iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$$

$$\therefore s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

at $s=1$ : $1^2-3(1)-4 = -2C$ $\quad C = 3$

at $s=3$ : $3^2-3(3)-4 = 4A$ $\quad A = -1$

at $s=0$ $\quad -4 = A + B(3) + C(-3)$

$$-4 = -1 + B(3) + (-9) \qquad B = 2$$

$$\therefore \mathcal{L}^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}\right\}$$

$$= -e^{3t} + 2e^{t} + 3te^{t}$$

(v) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+4+16}$$

$$\frac{s-5}{(s+2)(s+2)+16} = \frac{s-5}{(s+2)^2+4^2} = \frac{s}{(s+2)^2+4^2} - \frac{\cancel{4}}{4}\frac{5}{(s+2)^2+4^2}$$

$$= e^{-2t}\cos 4t - \frac{5}{4}e^{-2t}\sin 4t$$