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1 $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x)^2 y^{(2)} - 2xy^{(1)} + 2y = 0$

for $(1-x^2)y^2$

let $v = 1-x^2$ $\quad u = y^2$

$v' = -2x$ $\quad u^n = y^{(n+2)}$

$v'' = -2$ $\quad u^{(n-1)} = y^{(n+1)}$

$v''' = 0$ $\quad u^{(n-2)} = y^n$

$$y^n = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \frac{n(n-1)}{2!}y^n(-2)$$

$-2xy^{(1)}$

$v = -2x$ $\quad u = y^{(1)}$

$v' = -2$ $\quad u^{(n)} = y^{(n+1)}$

$u^{(n-1)} = y^n$

$$y^{(n+1)}(-2x) + ny^{(n)}(-2)$$

$2y$

$v = 2$ $\quad u = y$

$v' = 0$ $\quad u^{(n)} = y^{(n)}$

$= y^{(n)}(2)$

$$y^{(n)} = y^{(n+2)}(1-x^2) + ny^{(n+1)}(-2x) + \frac{n(n-1)}{2!}y^n(-2) + y^{(n+1)}$$

$(-2x) + ny^{(n)}(-2) + y^{(n)}(2)$

at $x = 0$

$$y^n = y^{(n+2)}(1) + ny^{(n+1)}(0) + \frac{n(n-1)}{2!}y^n(-2) + y^{(n+1)}(0) +$$

$ny^{(n)}(-2) + y^{(n)}(2)$

$$y^{(n)} = y^{(n+2)} - \frac{2n(n-1)}{2}y^n + ny^{(n)}(-2) + 2y^{(n)}$$

$$y^{(n)} = y^{(n+2)} - n(n-1)y^n + ny^n(-2) + 2y^{(n)}$$

$$y^{(n)} = y^{(n+2)} + y^{(n)}(1-n^2-2n+2)$$

$$y^{(n)} = y^{(n+2)} + y^{(n)}(-n^2-2n+3)$$

$$\cancel{y^{(n)}} \, y^{(n+2)} + y^{(n)}(-n^2-2n+3) = 0$$

$$y^{(n+2)} = -y^{(n)}(-n^2-2n+3)$$

$$y^{(n+2)} = y^{(n)}(n^2+2n-3)$$

when $n=0$

$$y^{(2)} = y^{(0)}(-3) = -3y^{0}$$

when $n=1$

$$y^{(3)} = y^{(1)}(1^2+2-3)$$

$$y^{(3)} = 0$$

when $n=2$

$$y^{(4)} = y^{(2)}(4+4-3)$$

$$y^{(4)} = 5y^{2} = (5)(-3)y^{0}$$

when $n=3$

$$y^{(5)} = y^{(3)}(9+6-3)$$

$$y^{(5)} = y^{(3)}(12) = 0$$

when $n=4$

$$y^{(6)} = y^{(4)}(16+8-3)$$

$$y^{(6)} = y^{(4)}(21) = (21)(5)(-3)y^{(0)}$$

when $n=5$

$$y^{(7)} = y^{(5)}(25+10-3)$$

$$y^{(7)} = y^{(5)}(32) = 0$$

when $n=6$

$$y^{(8)} = y^{(6)}(36+12-3)$$

$$y^{(8)} = y^{(6)}(45) = (45)(21)(5)(-3)y^{(0)}$$

For the Maclaurin Series,

$$y = y_0 + x(y^1)_0 + \frac{x^2}{2!}(y^2)_0 + \frac{x^3}{3!}(y^3)_0 + \frac{x^4}{4!}(y^{(4)})$$

$$y_0 + xy' + \frac{x^2}{2!}(-3y^{(0)}) + \frac{x^4}{4!}(5)(-3)y^{(0)} + \frac{x^6}{}(21)(5)(-3)y$$

$$y = y_0 + xy' + \frac{x^2}{2}(-3y^0) + \frac{x^4}{8}(-5)y^2 + \frac{x^6}{16} + \frac{x^8}{128}(-45y^0)\ldots$$

2. $3e^{-4t} - 5e^{4t}$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

ii $\sin 4t + \cos 4t$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

iii $t^2 + 2t^2 - t + 4$

$$\frac{2}{s^3} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

iv $e^{-2t}\cos 5t$

$$\frac{(s+2)}{(s+2)^2+5^2}$$

$$= \frac{(s+2)}{(s+2)^2+5^2}$$

$$= \frac{(s+2)}{(s+2)^2+25}$$

v $t \sin 3t$

$$L(\sin 3t) = \frac{3}{s^2+3^2}$$

$$L[t\sin 3t] = \frac{-d}{ds}(F(s))$$

$$= \frac{-d}{ds}\left(\frac{3}{s^2+3^2}\right)$$

$$= \frac{(s^2+3^2)(0)-(3)(2s)}{(s^2+9)^2}$$

$$= \frac{6s}{(s^2+9)^2}$$

$$= -\left[\frac{-6s}{(s^2+9^2)^2}\right]$$

vi $\frac{e^{-t} - e^{-2t}}{t}$

$L[e^{-t} - e^{-2t}]$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \left(\frac{1}{\sigma+1} - \frac{1}{\sigma+2}\right) d\sigma$$

$$\int_{s=\sigma}^{\infty} [\ln(\sigma+1) - \ln(\sigma+2)] d\sigma$$

$$= \ln\left(\frac{\infty+1}{\infty+2} - \ln\left(\frac{s+1}{s+2}\right)\right)$$

$$= \ln(1) - \ln\left(\frac{s+1}{s+2}\right)$$

$$= -\ln\left(\frac{s+1}{s+2}\right)$$

$$= \ln\left(\frac{s+1}{s+2}\right)^{-1}$$

$$= \ln\left(\frac{s+2}{s+1}\right)$$

vii $e^{4t} \cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

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$L[e^{4t}\cos 2t] = f(s-4)$

$= \frac{(s-4)}{(s-4)^2+4}$

ix $t^3 + 4t^2 + 5$

$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x $e^{3t}(t^2+4)$

$L[t^2+4) = \frac{2}{s^3} + \frac{4}{s}$

$L[e^{3t}(t^2+4)] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

xi $t^2 \cos t$

$L(\cos t] = \frac{s}{s^2+1}$

$L(t^2 \cos t) = \frac{d}{ds}\left(-\frac{d}{ds}\left(s/s^2+1\right)\right)$

$u = s \quad \frac{du}{dx} = 1$

$v = s^2+1 \quad \frac{dv}{dx} = 2s$

$L[t\cos t] = \frac{(s^2+1)(1) - s(2s)}{(s^2+1)^2}$

$= \frac{(s^2+1) - 2s^2}{(s^2+1)^2}$

$\frac{A+B}{A^2} = \frac{A(1+B/A)}{A^2}$

$\frac{1}{A} + \frac{B}{A^2}$

$= \frac{(s^2+1) - 2s^2}{(s^2+1)(s^2+1)}$

$= \frac{-s^2+1}{(s^2+1)(s^2+1)} \quad \frac{s^2+1}{(s^2+1)(s^2+1)} - \frac{2s^2}{(s^2+1)(s^2+1)}$

$-s^2+1$

$= \frac{-2s^2}{(s^2+1)}$

$\frac{-s^2+1}{(s^2+1)}$

$\frac{d}{ds}\left(\frac{-2s^2}{(s^2+1)}\right)$

$= \frac{(s^2+1)(-4s) - (-2s^2)(2s)}{(s^2+1)^2}$

$= -4s^3 - 4s + 4s^3$

$$= \frac{-4s}{(s^2+1)^2}$$

$$L\{t^2 \cos t\} = -4s/(s^2+1)^2.$$

x11 $\frac{\sinh 2t}{t}$

$$L[\sinh 2t] = 2/s^2-4$$

$$= \int_{\sigma=0}^{\infty} \frac{2}{\sigma^2-4} \, d\sigma$$

$$= 2\int_{\sigma=s}^{\infty} \frac{1}{\sigma^2-4}$$

$$2\int_{\sigma=s} \frac{-1}{-1} \frac{1}{\sigma-4} \, d\sigma$$

$$= -2\int_{\sigma=s}^{\infty} \frac{1}{2^2-\sigma} \, d\sigma$$

$$= -2\left(\tfrac{1}{2} \tan^{-1} \sigma/_2\right)^{\infty}$$

3i $\frac{S-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

at A, $s=3$, at B, $s=4$

For A

$(s-\cancel{3}) \times \frac{s-5}{(\cancel{s-3})(s-4)}$

at $s=3$

$\frac{(3-5)}{(3-4)}$

$= \frac{+2}{+1}$

$A = 2$

For B

$(\cancel{s-4}) \times \frac{s-5}{(s-3)(\cancel{s-4})}$

at $s=4$

$\frac{4-5}{4-3}$

$= \frac{-1}{1}$

$B = -1$

$F(s) = \frac{2}{s-3} + \frac{1}{s-4}$

$F(t) = 2e^{3t} + e^{4t}$

3ii $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

at A, $s=2$, at B, $s=4$

For A

$(\cancel{s-2}) \times \frac{2s-6}{(\cancel{s-2})(s-4)}$

at $s=2$

$\frac{2(2)-6}{2-4}$

$= \frac{4-6}{-2}$

$= \frac{-2}{-2}$

For B

$(\cancel{s-4}) \times \frac{2s-6}{(s-2)(\cancel{s-4})}$

at $\frac{2(4)-6}{4-2}$

$= \frac{8-6}{2}$

$= 2/2$

$= 1$

$F(s) = \frac{1}{s-2} + \frac{1}{s-4}$

$F(t) = e^{2t} + e^{4t}$

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

$5s-8 = As - 4A + Bs$

$A+B = 5$

$-4A = -8$

$A = -2$

$2 + B = 5$

$B = 5-2$

$B = 3$

$\frac{2}{s} + \frac{3}{s-4}$

$F(t) = 2 + 3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$\frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$\frac{(s-4)(s+1)}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$(s-4)(s+1) = A(s-1)^2 + B(s-1)(s-3) + C(s-1)^2$

$(s-4)(s+1) = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$(s^2-3s-4) = As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$A + B = 1$

$-2A - 4B + C = -3$

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$A = 1 - B$

$-2(1-B) - 4B + C = -3$; $-2B + 2B - 4B + C = -3$

$1 - B + 3B - 3C = 1$; $1 + 2B - 3C = 1$

$-2B + C = -1$

$+\ 2B - 3C = 0$

$0 - 2C = 1$

$C = -1/2$

$A + B = 1$

$-2A - 4B - 1/2 = -3$

$-2A - 4B = -3 + 1/2$

$-2A - 4B = -5/2$

$A = 1 - B$, $2A + 4B = 5/2$

$2(1 - B) + 4B = 5/2$

$\cancel{2A}\ 2 - 2B + 4B = 5/2$

$2B = 5/2 - 2$

$2B = 1/2$

$B = 1/4$

$A + 1/4 = 1$

$A = 3/4$

$\therefore \dfrac{\frac{3}{4}}{s-3} + \dfrac{1/4}{s-1} - \dfrac{1/2}{(s-1)^2}$

$f(t) = \frac{3}{4} e^{3t} + \frac{1}{4} e^{t} - \frac{1}{2} t e^{t}$

v $\dfrac{s-5}{s^2 + 4s + 20}$

$\dfrac{s-5}{s^2 + 4s + 20}$

$= \dfrac{s-5}{(s+2)^2 + 16}$

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$$\frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2-2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{2}{(s+2)^2+4^2} - \frac{5}{(s+2)^2+4^2}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2} \times \frac{4}{4}$$

$$\frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\frac{\times 4}{(s+2)^2+4^2}$$

$$= e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$