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15/ENG06/068
Mechanical Engineering

ENG 381 Assignment 4

1) $(1-x^2)^{dy}/dx - 2x\,{}^{dy}/dx + 2y = 0$

$(1-x^2)y^2 - 2xy' + 2y = 0$

$y^n = U^n V + nV^{n-1}V' + \frac{n(n-1)}{2!}U^{n-2}V^2 + \cdots$

$\left[y^{(2+n)} \cdot (1-x^2) + ny^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!}y^n \cdot (-2)\right] + \left[y^{(1+n)} \cdot -2x + ny^n\right]_2 + 2y^n = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

let $x = 0$

$\therefore$ $y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^n = 0$

$y^{(n+2)} + y^n[-n(n+1) - 2n + 2] = 0$

$y^{n+2} = -(y)^n_0[-n^2 - n + 2]$

For $n=0$ $\quad y^2 = -y^0 \cdot 2 = -2y^0$

for $n=1$ $\quad y^3 = 0$

for $n=2$ $\quad y^4 = -y^2(-4) = -8y^0$

$n=3$ $\quad y^5 = 0$

$n=4$ $\quad y^6 = -2y^0$

$n=5$ $\quad y^7 = 0$

$y = y^0 + xy' + {}^{x^2}/_{2!}y^2 + {}^{x^3}/_{3!}y^3 + \cdots$

$y = y^0 + xy' + {}^{x^2}/_{2!}(-2)y^0 + {}^{x^3}/_{3!}(0) + {}^{x^4}/_{4!}(-8)(-2)y^0 + \cdots$

$y = y^0[1 - x^2 - {}^{x^4}/_3 - {}^{x^6}/_5] + y'_0[x]$

2) i) $3e^{-4t} - 5e^{4t}$

$L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$= 3L[e^{-4t}] - 5L[e^{4t}]$

$= 3\left[\frac{1}{s-4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s+4}$

ii) $\sin 4t + \cos 4t$

$L[\sin 4t] + L[\cos 4t]$

$= a/s^2+a^2 + s/s^2+a^2$

$= 4/s^2+16 + s/s^2+16$

$= 4+s/(s^2+16)$

iii) t^3+2t^2-t+4

$L[t^3]+L[2t^2] - L[t]+L[4]$

$t^n = n!/s^{n+1}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$

$= 6/s^4 + 4/s^3 - 1/s^2 + 4/s$

iv) $e^{-2t}\cos 5t$

$L[\cos 5t] = s/s^2+a^2$

$= s/s^2+25$

$L[e^{-2t}\cos 5t] = s+2 / (s+2)^2+25$

v) $t\sin 3t$

$L[\sin 3t] = a/s^2+a^2$

$= 3/s^2+3^2$

$F(s) = 3/s^2+9$

$L[t\sin 3t] = -f'(s)$

Using quotient rule

$\frac{V\,du/ds - U\,dv/ds}{V^2} = \frac{[s^2+9]\cdot 0 - 3\cdot[2s]}{[s^2+9]^2}$

$= \frac{-6s}{[s^2+9]^2}$

$= \frac{6s}{[s^2+9]^2}$

vi) $\frac{e^{-t}-e^{-2t}}{t}$

$$L\left[\frac{e^{-t}-e^{-2t}}{t}\right]$$

$$\lim_{t\to 0}\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \frac{1-1}{0} = \frac{0}{0} \Rightarrow \text{Indeterminate}$$

Applying L'Hospital's Rule

$$\lim_{t\to 0}\left[\frac{-e^{-t}-(-2)e^{-2t}}{1}\right] = \left[\frac{-1+2}{1}\right] = 1 \Rightarrow \text{determinate}$$

$$L\left[f(t)/t\right] = \int_{t=s}^{\infty} f_s \, ds$$

$$f_s = L[f(t)]$$

$$L = [f(t)] = e^{-t} - e^{-2t} = 1/s+a - 1/s+a$$

$$= 1/s+1 - 1/s+2$$

$$F_\epsilon = 1/\epsilon+1 - 1/\epsilon+2$$

$$\int f_\epsilon \quad L\left[\frac{f(t)}{t}\right] = \int_{\epsilon=s}^{\infty} 1/\epsilon+1 - 1/\epsilon+2 \, d\epsilon$$

$$= \left[\ln[\epsilon+1] - \ln[\epsilon+2]\right]_s^{\infty}$$

$$= \left[\ln[\epsilon+1) - \ln(\epsilon+2]\right]_s^{\infty}$$

$$= \left[\ln\frac{\epsilon+1}{(\epsilon+2)}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

vii) $e^{4t}\cos 2t$

$$L(e^{st}\cos 2t] = e^{4t} L[\cos 2t]$$

$$L[\cos 2t] = s/s^2+2^2$$

$$= s/s^2+4$$

$$L[e^{4t}\cos 2t] = s-4/(s-4)^2+4$$

viii) $t\sin 2t$

$$\exists L[t\sin 2t] = -d/ds\,[f(s)]$$

$$f(s) = L[\sin 2t] = a/s^2+2^2$$

$$f(s) = 2/s^2+4$$

$f'(s)$ = using quotient Rule

$$\frac{v\,du/ds - u\,dv/ds}{v^2}$$

$$f'(s) = \frac{-4s}{[s^2+4]^2}$$

$$\therefore L[t\sin 2t] = -f'(s)$$

$$= -1 = \left[\frac{-4s}{[s^2+4]^2}\right]$$

$$= \frac{4s}{[s^2+4]^2}$$

ix) t^3+4t^2+5

$$L[t^3] + 4L[t^2] + L[5]$$

$$= 3!/s^{3+1} + 4\left[2!/s^{2+1}\right] + 5/s$$

$$= 6/s^4 + 8/s^3 + 5/s$$

x) $e^{3t}(t^2+4)$

let $x = t^2+4$

$$L[e^{3t}x]$$

$$L[x] = L[t^2+4]$$

$$= L[t^2] + L[4]$$

$$= 2!/s^{2+1} + 4/s$$

$$= 2/s^3 + 4/s$$

$$L[e^{3t}x] = \frac{2}{[s-3]^3} + \frac{4}{[s-3]}$$

2i) $t^2 \cos t$

$L[t^2 \cos t] = t^2 L[\cos t]$

$f(s) = L[\cos t] = s/s^2+1^2$

$f(s) = s/s^2+1^2$

$f'(s)$ using quotient rule

$f'(s) =$

3) i) $\frac{s-5}{(s-3)(s-4)}$

$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = A/(s-3) + B/(s-4)$

$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$

Assuming $s=4$

$4-5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s=3$

$3-5 = A(3-4) + B(3-3)$

$-2 = -A$

$A = 2$

$\therefore \frac{s-5}{(s-3)(s-4)} = \frac{2}{(s-3)} + \frac{-1}{(s-4)}$

$= 2e^{3t} - e^{4t}$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$

$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$2s-6 = A(s-4) + B(s-2)$

Assuming $s=4$

$(2(4)-6) = A(4-4) + B(4-2)$

$8-6 = A(0) + B(2)$

$2 = 2B$

$B = 1$

Assuming $s=2$

$2(2)-6 = A(2-4) + B(2-2)$

$-2 = -2A$

$A = 1$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$$

iii) $\dfrac{5s-8}{s(s-4)}$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4)-8 = A(4-4) + B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

Assuming $s=0$

$5(0)-8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

iv) $\frac{s-5}{s^2+4s+20}$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right]$$

$$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7\times 4/4}{(s+2)^2+4^2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4}\cdot\frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$

v) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$$\Rightarrow f(s) = \frac{A}{s+3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A = \left.\frac{s^2-3s-4}{(s-1)^2}\right|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$$

$$B = \left.\frac{s^2-3s-4}{(s-3}\right|_{s-1} = \frac{1^2-3(1)-4}{1-3} = 3$$

$$C = \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-[s^2-3s-4]}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3te^{t} + 2e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$

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