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Computer Adu

(1) dy/dt + 3y = e^{-3t}

t=0, y=2

S(y)s - y(0) + 3y(s) = 1/(s+2)

S(y)s - 2 + 3y(s) = 1/(s+2)

y(s)(s+3) = 2/(s+2) + 1/(s+2)

y(s)(s+3) = (2s+5)/(s+2)

y(s) = (2s+5)/((s+3)(s+2)) = A/(s+3) + B/(s+2)

2s+5 = A(s+2) + B(s+3)

s = -2

2(-2)+5 = B(-2+3)

B = 1

s = -3

2(-3)+5 = A(-3+2)

A = 1

y(s) = (s^2+4) + 2 / ((s-6)(s+2)^2) = A(s+2)^2 + B(s-6)(s+2) + C

(s^2+4)+2 = A(s+2)^2 + B(s-6)(s+2) + C(s-6)

s = 6

42/64 = 64A/6A

A = 21/32

s = 0

6 = 4A - 12B - 6C

6 = 4(21/32) - 12B - 6(-1/4)

6 = 84/32 + 30/4 - 12B

6 = 81/8 - 12B

6 - 81/8 = -12B

-33/8 = B

B = 11/32

(2) 3 dy/dt - 6y = sin 2t

t=0

3(dy(s) - y(0)) - 6y(s) = 2/(s^2+4)

3(8y(s) - 1) - 6y(s) = 2/(s^2+4)

3y(s)(s-6) = (s+2)^2 + 2

y(s) = (s^2+4) + 2 / ((s-6)(s+2)^2)

A/(s-6) + B/(s+2) + C/(s+2)^2

s = -2

10 = -8C

C = -5/4

= 21/32 (1/(s-6)) + 11/32 (1/(s+2)) = 5/4 (1/(s^2+4))

= 21/32 e^{6t} + 11/32 e^{-2t} - 5/4 sin 2t

$$(3) \frac{dy}{dt} = 4y = 8$$

$$t=0, y=2$$

$$\mathcal{L}y(s) - y(0) = 4y(s) = \frac{8}{s}$$

$$\mathcal{L}y(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8}{s} + \frac{2}{1} = \frac{2s+8}{s}$$

$$y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$= \frac{A(s-4) + Bs}{s(s-4)}$$

$$= 2s+8 = A(s-4) + Bs$$

$$s=0$$

$$2(0)+8 = A(0-4) + B(0)$$

$$\frac{8}{-4} = \frac{-4A}{-4}$$

$$A = -2$$

$$s=4$$

$$2(4)+8 = 4B$$

$$\frac{16}{4} = B$$

$$B = 4$$

$$= \frac{-2}{s} + \frac{4}{s-4}$$

$$= \mathcal{L}^{-1}\left(\frac{-2}{s}\right) + \mathcal{L}^{-1}\left(\frac{4}{s-4}\right)$$

$$= \underline{\underline{4e^{4t} - 2}}$$

$$(4) \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{2t}$$

$$t=0, y=0, y'=2$$

$$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 0 - 2 - 6sy(s) + 12 + 8y(s) = \frac{1}{s-2}$$

$$s^2y(s) - 6sy(s) + 8y(s) + 10 = \frac{1}{s-2}$$

$$y(s)(s^2 - 6s + 8) = \frac{10}{1} + \frac{1}{s-2} = \frac{-10s+31}{s-2}$$

$$y(s) = \frac{-10s+31}{(s^2-6s+8)(s-2)} = \frac{-10s+31}{(s-2)(s-4)(s-3)}$$

$$y(s) = \frac{-10s+31}{(s^2-6s+8)(s-3)}$$

$$\frac{A}{s-2} + \frac{B}{s-4} + \frac{C}{s-3}$$

$$= -10s+31 = A(s-4)(s-3) + B(s-2)(s-3) + C(s-2)(s-4)$$

$$s=2$$

$$-10(2)+31 = A(2-4)(2-3)$$

$$A = \frac{11}{2}$$

$$s=4$$

$$-10(4)+31 = B(4-2)(4-3), B = \frac{-9}{2}$$

$$s=3$$

$$-10(3)+31 = C(3-2)(3-4), C = -1$$

$$= \frac{1}{2} \left(\frac{11}{s-2} - \frac{9}{s-4} \right) - \frac{1}{s-3}$$

$$= 2^{-1} (11e^{2t} - 9e^{4t}) - e^{3t}$$