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ASSIGNMENT 4

$$1) (1-x^2) \frac{d^2y}{dx^2} - 2xy \frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' - 2xy' + 2y = 0$$

$$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y^{(0)} = 0$$

$w_1 \quad w_2 \quad w_3$

For $w_1; (1-x^2)y^{(2)}$

$$u = y^{(2)} \quad v = (1-x^2)$$
$$u^{(1)} = y^{(3)} \quad v^{(1)} = -2x$$
$$u^{(2)} = y^{(4)} \quad v^{(2)} = -2$$

For $w_2; -2xy^{(1)}$

$$u = y^{(1)} \quad v = -2x$$
$$u^{(1)} = y^{(2)} \quad v^{(1)} = -2$$
$$u^{(2)} = y^{(3)} \quad v^{(2)} = -2$$

For $w_3; 2y^{(0)}$

$$u = y^{(0)} \quad v = 2$$
$$u^{(1)} = y^{(1)} \quad v^{(1)} = 2$$

$$w_1 \Rightarrow (1-x^2)y^{(2)} = (1-x^2)y^{(2+2)} - n2xy^{(2+1)} - \frac{n(n-1)}{2!}2y^{(2)}$$
$$= (1-x^2)y^{(n+2)} - n2xy^{(n+1)} - n(n-1)y^{(n)}$$

$$w_2 \Rightarrow -2xy^{(n+1)} + 2ny^{(n)}$$

$$w_3 \Rightarrow 2y^{(n)}$$

The equation then becomes

$$(1-x^2)y^{(n+2)} - n2xy^{(n+1)} - n(n-1)y^{(n)} - 2xy^{(n+1)} - 2ny^{(n)} + 2y^{(n)} = 0$$

when $x=0$

$$(y^{(n+2)})_0 - n(n-1)(y^{(n)})_0 - 2n(y^{(n)})_0 + 2(y^{(n)})_0 = 0$$

$$(y^{(n+2)})_0 - (n^2+n)(y^{(n)})_0 - 2n(y^{(n)})_0 + 2(y^{(n)})_0 = 0$$

$$(y^{(n+2)})_0 = (n^2+n)(y^{(n)})_0 + 2n(y^{(n)})_0 - 2(y^{(n)})_0$$

$$(y^{(n+2)})_0 = (y^{(n)})_0 [n^2+n+2n-2]$$

$$n=0; (y^{(2)})_0 = -2(y^{(0)})_0$$

$$n=1; (y^{(3)})_0 = 2(y^{(1)})_0$$

$$n=2; (y^{(4)})_0 = 8(y^{(2)})_0 = 8(-2)(y^{(0)})_0 = -16(y^{(0)})_0$$

$$n=3; (y^{(5)})_0 = 16(y^{(3)})_0 = 16(2)(y^{(1)})_0 = 32(y^{(1)})_0$$

$$n=4; (y^{(6)})_0 = 26(y^{(4)})_0 = 26(8)(-2)(y^{(0)})_0 = -416(y^{(0)})_0$$

$$n=5; (y^{(7)})_0 = 38(y^{(5)})_0 = 38(16)(2)(y^{(1)})_0 = 1216(y^{(1)})_0$$

$$n=6; (y^{(8)})_0 = 52(y^{(6)})_0 = 52(26)(8)(-2)(y^{(0)})_0 = -421632(y^{(0)})_0$$

$$n=7; (y^{(9)})_0 = 68(y^{(7)})_0 = 68(38)(16)(2)(y^{(1)})_0 = 82688(y^{(1)})_0$$

$$y = (y^{(0)})_0 + x (y^{(1)})_0 + \frac{x^2}{2!} (y^{(2)})_0 + \frac{x^3}{3!} (y^{(3)})_0 + \frac{x^4}{4!} (y^{(4)})_0 + \frac{x^5}{5!} (y^{(5)})_0 \\ + \frac{x^6}{6!} (y^{(6)})_0 + \frac{x^7}{7!} (y^{(7)})_0 + \frac{x^8}{8!} (y^{(8)})_0 + \frac{x^9}{9!} (y^{(9)})_0 + \dots$$

$$y = (y^{(0)})_0 + x (y^{(1)})_0 - \frac{2x^2}{2!} (y^{(2)})_0 + \frac{2x^3}{3!} (y^{(3)})_0 - \frac{16x^4}{4!} (y^{(4)})_0 + \frac{32x^5}{5!} (y^{(5)})_0 \\ - \frac{416x^6}{6!} (y^{(6)})_0 + \frac{1216x^7}{7!} (y^{(7)})_0 - \frac{21632x^8}{8!} (y^{(8)})_0 + \frac{82688x^9}{9!} (y^{(9)})_0 + \dots$$

$$y = (y^{(0)})_0 + x (y^{(1)})_0 - x^2 (y^{(2)})_0 + \frac{1}{3} x^3 (y^{(3)})_0 - \frac{2}{3} x^4 (y^{(4)})_0 + \frac{4}{5} x^5 (y^{(5)})_0 \\ - \frac{26}{45} x^6 (y^{(6)})_0 + \frac{76}{315} x^7 (y^{(7)})_0 - \frac{169}{315} x^8 (y^{(8)})_0 + \frac{646}{2835} x^9 (y^{(9)})_0 + \dots$$

$$\therefore y = (y^{(0)})_0 \left[1 - x^2 + \frac{2x^4}{3} - \frac{26x^6}{45} - \frac{169x^8}{315} + \dots \right] \\ = (y^{(1)})_0 \left[x + \frac{x^3}{3} + \frac{4x^5}{15} + \frac{76x^7}{315} + \frac{646x^9}{2835} + \dots \right]$$

$$2i) \mathcal{L}\{3e^{-4t} - 5e^{4t}\} = 3\mathcal{L}\{e^{-4t}\} - 5\mathcal{L}\{e^{4t}\} = \frac{3}{s+4} - \frac{5}{s-4}$$

$$ii) \mathcal{L}\{\sin 4t + \cos 4t\} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$$

$$iii) \mathcal{L}\{t^3 + 2t^2 - t + 4\} \\ = \mathcal{L}\{t^3\} + 2\mathcal{L}\{t^2\} - \mathcal{L}\{t\} + \mathcal{L}\{4\} \\ = \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

$$iv) \mathcal{L}\{e^{-2t} \cos 5t\} = \frac{s+2}{(s+2)^2 + 25}$$

$$v) \mathcal{L}\{t \sin 3t\}$$

$$\text{sol}$$

$$\mathcal{L}\{t \sin 3t\} = \frac{3}{s^2 + 9}$$

$$\mathcal{L}\{t \sin 3t\} = -\frac{d}{ds} \left(\frac{3}{s^2 + 9} \right)$$

$$\text{Solving } \frac{d}{ds} \left(\frac{3}{s^2 + 9} \right)$$

$$\text{Using Quotient rule } \left[\frac{vdu - udv}{v^2} \right]$$

$$u = 3 \quad v = s^2 + 9$$

$$du = 0 \quad dv = 2s$$

$$\frac{d}{ds} \left(\frac{3}{s^2 + 9} \right) = \frac{(s^2 + 9) \cdot 0 - 3(2s)}{(s^2 + 9)^2} = \frac{-6s}{(s^2 + 9)^2}$$

$$\mathcal{L}\{t \sin 3t\} = -\frac{d}{ds} \left(\frac{3}{s^2 + 9} \right) = \frac{6s}{(s^2 + 9)^2}$$

$$vi) \mathcal{L}\left\{ \frac{e^{-t} - e^{-2t}}{t} \right\}$$

$$\text{sol}$$

$$\lim_{t \rightarrow 0} \left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \frac{e^{-0} - e^{-2(0)}}{0} = \frac{1-1}{0} = \frac{0}{0}$$

Using L'Hopital's rule

$$\lim_{t \rightarrow 0} \left\{ \frac{-e^{-t} + 2e^{-2t}}{1} \right\} = \frac{-e^{-0} + 2e^{-2(0)}}{1} = -1 + 2 = 1$$

$$\mathcal{L}\left\{ \frac{e^{-t} - e^{-2t}}{t} \right\}$$

$$\mathcal{L}\{e^{-t} - e^{-2t}\} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\mathcal{L}\left\{ \frac{e^{-t} - e^{-2t}}{t} \right\} = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} d\sigma$$

$$= \left[\ln(\sigma+1) - \ln(\sigma+2) \right]_s^{\infty}$$

$$= \left[\ln \left(\frac{s+1}{s+2} \right) \right]_s^{\infty}$$

$$= \ln \left[\frac{\infty+1}{\infty+2} \right] - \ln \left[\frac{s+1}{s+2} \right]$$

$$= \ln 1 - \ln \left(\frac{s+1}{s+2} \right) = 0 - \ln \left(\frac{s+1}{s+2} \right) = -\ln \left(\frac{s+1}{s+2} \right)$$

vii) $L\{e^{4t} \cos 2t\} = \frac{s-4}{(s-4)^2 + 4}$

viii) $L\{t \sin 2t\}$

sol

$$L\{\sin 2t\} = \frac{2}{s^2 + 4}$$

$$L\{t \sin 2t\} = -\frac{d}{ds} \left(\frac{2}{s^2 + 4} \right)$$

Solving $\frac{d}{ds} \left(\frac{2}{s^2 + 4} \right)$

Using Quotient rule $\left[\frac{vdu - u dv}{v^2} \right]$

$$u = 2 \quad v = s^2 + 4$$

$$du = 0 \quad dv = 2s$$

$$\frac{d}{ds} \left(\frac{2}{s^2 + 4} \right) = \frac{(s^2 + 4) \cdot 0 - 2(2s)}{(s^2 + 4)^2} = \frac{-4s}{(s^2 + 4)^2}$$

$$L\{t \sin 2t\} = -\frac{d}{ds} \left(\frac{2}{s^2 + 4} \right) = \frac{4s}{(s^2 + 4)^2}$$

ix) $L\{t^3 + 4t^2 + 5\}$

$$= L\{t^3\} + 4L\{t^2\} + L\{5\}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$\begin{aligned}
 x) \quad & L\{e^{3t}(t^2+4)\} \\
 &= L\{t^2 e^{3t}\} + L\{4e^{3t}\} \\
 &= \frac{2}{(s-3)^2} + \frac{4}{s-3}
 \end{aligned}$$

$$xi) \quad L\{t^2 \cos t\}$$

sol

$$L\{\cos t\} = \frac{s}{s^2+1}$$

$$-\frac{d}{ds} \left(\frac{s}{s^2+1} \right) = - \left(\frac{(s^2+1) \cdot 1 - s(2s)}{(s^2+1)^2} \right) = - \left(\frac{s^2+1-2s^2}{(s^2+1)^2} \right) = - \left(\frac{1-s^2}{(s^2+1)^2} \right) = \frac{s^2-1}{(s^2+1)^2}$$

$$L\{t^2 \cos t\} = -\frac{d}{ds} \left(-\frac{d}{ds} \left(\frac{s}{s^2+1} \right) \right) = -\frac{d}{ds} \left(\frac{s^2-1}{(s^2+1)^2} \right)$$

$$-\frac{d}{ds} \left(\frac{s^2-1}{(s^2+1)^2} \right) = - \left(\frac{(s^2+1)^2 \times 2s - (s^2-1) \cdot 4s(s^2+1)}{(s^2+1)^4} \right)$$

$$= - \left[\frac{2s(s^2+1)^2 - 4s(s^2-1)(s^2+1)}{(s^2+1)^4} \right]$$

$$= - \frac{(s^2+1) [2s(s^2+1) - 4s(s^2-1)]}{(s^2+1)^4}$$

$$= - \frac{2s(s^2+1 - 2s^2 + 2)}{(s^2+1)^3}$$

$$= \frac{2s(-s^2 - 1 + 2s^2 - 2)}{(s^2+1)^3}$$

$$u = s^2 - 1 \quad v = (s^2+1)^2$$

$$du = 2s \quad dv = 4s(s^2+1)$$

solving for dv

$$(s^2+1)^2$$

$$u = s^2 + 1 \quad ; \quad y = u^2$$

$$\frac{du}{ds} = 2s \quad \frac{dy}{du} = 2u$$

$$\frac{dy}{ds} = \frac{dy}{du} \times \frac{du}{ds}$$

$$= 2u \times 2s$$

$$= 4su = 4s(s^2+1)$$

$$\text{xii) } L \left\{ \frac{\sinh 2t}{t} \right\}$$

sol

$$\lim_{t \rightarrow 0} \left\{ \frac{\sinh 2t}{t} \right\} = \frac{\sinh 2(0)}{0} = \frac{0}{0}$$

Using L'Hopital's rule

$$\lim_{t \rightarrow 0} \left\{ \frac{2 \cosh 2t}{1} \right\} = \frac{2 \cosh 2(0)}{1} = 2 \times 1 = 2$$

$$L \left\{ \frac{\sinh 2t}{t} \right\}$$

$$L \{ \sinh 2t \} = \frac{2}{s^2 - 4}$$

$$L \left\{ \frac{\sinh 2t}{t} \right\} = \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2 - 4} d\sigma = \int_{\sigma=s}^{\infty} \frac{2}{(\sigma-2)(\sigma+2)} d\sigma$$

$$\frac{2}{(\sigma-2)(\sigma+2)} = \frac{A}{\sigma-2} + \frac{B}{\sigma+2} = \frac{A(\sigma+2) + B(\sigma-2)}{(\sigma-2)(\sigma+2)}$$

$$2 = A(\sigma+2) + B(\sigma-2)$$

when $\sigma = 2$

$$2 = A(2+2)$$

$$2 = 4A$$

$$A = \frac{2}{4} = \frac{1}{2}$$

when $\sigma = -2$

$$2 = B(-2-2)$$

$$2 = -4B$$

$$B = \frac{-2}{4} = -\frac{1}{2}$$

$$\therefore \int_{\sigma=s}^{\infty} \frac{2}{(\sigma-2)(\sigma+2)} d\sigma = \int_{\sigma=s}^{\infty} \frac{1/2}{\sigma-2} d\sigma - \int_{\sigma=s}^{\infty} \frac{1/2}{\sigma+2} d\sigma$$

$$\Rightarrow \frac{1}{2} \int_{\sigma=s}^{\infty} \frac{1}{\sigma-2} d\sigma - \frac{1}{2} \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2} d\sigma$$

$$\Rightarrow \frac{1}{2} \left[\ln(\sigma-2) - \ln(\sigma+2) \right]_s^{\infty}$$

$$= \frac{1}{2} \left[\ln \left(\frac{s-2}{s+2} \right) \right]_s^{\infty}$$

$$= \frac{1}{2} \left[\ln \left(\frac{\infty-2}{\infty+2} \right) - \ln \left(\frac{s-2}{s+2} \right) \right]$$

$$= \frac{1}{2} \left[\ln 1 - \ln \left(\frac{s-2}{s+2} \right) \right]$$

$$= \frac{1}{2} \left[0 - \ln \left(\frac{s-2}{s+2} \right) \right] = \underline{\underline{-\frac{1}{2} \ln \left(\frac{s-2}{s+2} \right)}}$$

$$3i) \mathcal{L}^{-1} \left\{ \frac{s-5}{(s-3)(s-4)} \right\}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

$$s-5 = A(s-4) + B(s-3)$$

when $s=3$

$$3-5 = A(3-4)$$

$$-2 = -A$$

$$A = 2$$

when $s=4$

$$4-5 = B(4-3)$$

$$-1 = B$$

$$B = -1$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$\mathcal{L}^{-1} \left\{ \frac{s-5}{(s-3)(s-4)} \right\} = \mathcal{L}^{-1} \left\{ \frac{2}{s-3} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s-4} \right\}$$

$$= 2e^{3t} - e^{4t}$$

$$ii) \frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4} = \frac{A(s-4) + B(s-2)}{(s-2)(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

when $s=2$

$$2(2)-6 = A(2-4)$$

$$-2 = -2A$$

$$A = 1$$

when $s=4$

$$2(4)-6 = B(4-2)$$

$$2 = 2B$$

$$B = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$\begin{aligned} L^{-1} \left\{ \frac{2s-6}{(s-2)(s-4)} \right\} &= L^{-1} \left\{ \frac{1}{s-2} \right\} + L^{-1} \left\{ \frac{1}{s-4} \right\} \\ &= \underline{\underline{e^{2t} + e^{4t}}} \end{aligned}$$

$$\text{iii) } \frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4} = \frac{A(s-4) + B(s)}{s(s-4)}$$

$$5s-8 = A(s-4) + B(s)$$

when $s=0$

$$5(0)-8 = A(0-4)$$

$$-8 = -4A$$

$$A = \frac{-8}{-4} = 2$$

when $s=4$

$$5(4)-8 = B(4)$$

$$12 = 4B$$

$$B = \frac{12}{4} = 3$$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$\begin{aligned} L^{-1} \left\{ \frac{5s-8}{s(s-4)} \right\} &= L^{-1} \left\{ \frac{2}{s} \right\} + L^{-1} \left\{ \frac{3}{s-4} \right\} \\ &= \underline{\underline{2 + 3e^{4t}}} \end{aligned}$$

$$\text{iv) } \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{s-1}$$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2}{(s-3)(s-1)^2} + \frac{B(s-3)(s-1)}{(s-3)(s-1)^2} + \frac{C(s-3)(s-1)}{(s-3)(s-1)^2}$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)(s-1)$$

when $s=3$

$$(3)^2 - 3(3) - 4 = A(3-1)^2$$

$$-4 = 4A$$

$$A = -1$$

$$B = C = 0$$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{-1}{s-3}$$

$$L^{-1} \left\{ \frac{s^2 - 3s - 4}{(s-3)(s-1)^2} \right\} = L^{-1} \left\{ \frac{-1}{s-3} \right\}$$

$$= \underline{\underline{-e^{3t}}}$$

v) ~~s-5~~ $\frac{s-5}{s^2 + 4s + 20}$

sol

$$\frac{s-5}{s^2 + 4s + 20} = \frac{(s+2) - 7}{(s+2)^2 + 16}$$

$$L^{-1} \left\{ \frac{(s+2) - 7}{(s+2)^2 + 16} \right\} = L^{-1} \left\{ \frac{(s+2)}{(s+2)^2 + 16} \right\} - 7L^{-1} \left\{ \frac{1}{(s+2)^2 + 16} \right\}$$

$$= \cos 4t e^{-2t} - 7 \sin 4t e^{-2t}$$