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ASSIGNMENT 4

(1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$\Rightarrow (1-x^2)y'' - 2xy' + 2y = 0$

$\Rightarrow \underbrace{(1-x^2)y^{(2)}}_{w1} - \underbrace{2xy^{(1)}}_{w2} + \underbrace{2y^{(0)}}_{w3} = 0$

for $w1$; $(1-x^2)y^{(2)}$

$u = y^{(2)}$ $\quad v = (1-x^2)$
$u^{(n)} = y^{(n+2)}$ $\quad v^{(0)} = (1-x^2)$
$u^{(n-1)} = y^{(n+1)}$ $\quad v^{(1)} = -2x$
$u^{(n-2)} = y^{(n)}$ $\quad v^{(2)} = -2$

For $w2$; $-2xy^{(1)}$

$u = y^{(1)}$ $\quad v = -2x$
$u^{(n)} = y^{(n+1)}$ $\quad v^{(0)} = -2x$
$u^{(n-1)} = y^{(n)}$ $\quad v^{(1)} = -2$

For $w3$; $2y^{(0)}$

$u = y^{(0)}$ $\quad v = 2$
$u^{(n)} = y^{(n)}$ $\quad v^{(0)} = 2$

$w1 \Rightarrow (1-x^2)y^{(2)} = (1-x^2)y^{(n+2)} - n2xy^{(n+1)} - \frac{n(n-1)}{\cancel{2!}}\cancel{2}y^{(n)}$

$= (1-x^2)y^{(n+2)} - n2xy^{(n+1)} - n(n-1)y^{(n)}$

$w2 \Rightarrow -2xy^{(n+1)} - 2ny^{(n)}$

$w3 \Rightarrow 2y^{(n)}$

The equation then becomes

$(1-x^2)y^{(n+2)} - n\cancel{2x}y^{(n+1)} - n(n-1)y^{(n)} - \cancel{2x}y^{(n+1)} - 2ny^{(n)} + 2y^{(n)} = 0$

when $x=0$

$(y^{(n+2)})_0 - n(n-1)(y^{(n)})_0 - 2n(y^{(n)})_0 + 2(y^{(n)})_0 = 0$

$(y^{(n+2)})_0 - (n^2+n)(y^{(n)})_0 - 2n(y^{(n)})_0 + 2(y^{(n)})_0 = 0$

$(y^{(n+2)})_0 = (n^2+n)(y^{(n)})_0 + 2n(y^{(n)})_0 - 2(y^{(n)})_0$

$(y^{(n+2)})_0 = (y^{(n)})_0[(n^2+n) + 2n - 2]$

$n=0$: $(y^{(2)})_0 = -2(y^{(0)})_0$

$n=1$: $(y^{(3)})_0 = 2(y^{(1)})_0$

$n=2$: $(y^{(4)})_0 = 8(y^{(2)})_0 = 8(-2)(y^{(0)})_0 = -16(y^{(0)})_0$

$n=3$: $(y^{(5)})_0 = 16(y^{(3)})_0 = 16(2)(y^{(1)})_0 = 32(y^{(1)})_0$

$n=4$: $(y^{(6)})_0 = 26(y^{(4)})_0 = 26(8)(-2)(y^{(0)})_0 = -416(y^{(0)})_0$

$n=5$: $(y^{(7)})_0 = 38(y^{(5)})_0 = 38(16)(2)(y^{(1)})_0 = 1216(y^{(1)})_0$

$n=6$: $(y^{(8)})_0 = 52(y^{(6)})_0 = 52(26)(8)(-2)(y^{(0)})_0 = -21632(y^{(0)})_0$

$n=7$: $(y^{(9)})_0 = 68(y^{(7)})_0 = 62(38)(16)(2)(y^{(1)})_0 = 82688(y^{(1)})_0$

(iv) $L\{e^{-2t}\cos 5t\} = \dfrac{s+2}{(s+2)^2+25}$ //

(v) $L\{t \sin 3t\}$

Soln

$L\{\sin 3t\} = \dfrac{3}{s^2+9}$

$L\{t\sin 3t\} = -\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right)$

Solving $\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right)$

Using Quotient rule $\left[\dfrac{vdu - udv}{v^2}\right]$

$u = 3 \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right) = \dfrac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = \dfrac{-6s}{(s^2+9)^2}$

$L\{t\sin 3t\} = -\dfrac{d}{ds}\left(\dfrac{3}{s^2+9}\right) = \dfrac{6s}{(s^2+9)^2}$ //

(vi) $L\left\{\dfrac{e^{-t} - e^{-2t}}{t}\right\}$

Soln

$\lim_{t \Rightarrow 0}\left\{\dfrac{e^{-t}-e^{-2t}}{t}\right\} = \dfrac{e^{-(0)} - e^{-2(0)}}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$

Using L'Hopital's rule

$\lim_{t \Rightarrow 0}\left\{\dfrac{-e^{-t} + 2e^{-2t}}{1}\right\} = \dfrac{-e^{-(0)} + 2e^{-2(0)}}{1} = -1 + 2 = 1$

$L\left\{\dfrac{e^{-t} - e^{-2t}}{t}\right\}$

$L\{e^{-t} - e^{-2t}\} = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \frac{x^4}{4!}(y^{(4)})_0 + \frac{x^5}{5!}(y^{(5)})_0 + \frac{x^6}{6!}(y^{(6)})_0$$

$$+ \frac{x^7}{7!}(y^{(7)})_0 + \frac{x^8}{8!}(y^{(8)})_0 + \frac{x^9}{9!}(y^{(9)})_0 + \dots$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 - \frac{2x^2}{2!}(y^{(0)})_0 + \frac{2x^3}{3!}(y^{(1)})_0 - \frac{16x^4}{4!}(y^{(0)})_0 + \frac{32x^5}{5!}(y^{(1)})_0$$

$$- \frac{416x^6}{6!}(y^{(0)})_0 + \frac{1216x^7}{7!}(y^{(1)})_0 - \frac{21632x^8}{8!}(y^{(0)})_0 + \frac{82688x^9}{9!}(y^{(1)})_0 + \dots$$

$$y = (y^{(0)})_0 + x(y^{(1)})_0 - x^2(y^{(0)})_0 + \frac{1}{3}x^3(y^{(1)})_0 - \frac{2}{3}x^4(y^{(0)})_0 + \frac{4}{5}x^5(y^{(1)})_0$$

$$- \frac{26}{45}x^6(y^{(0)})_0 + \frac{76}{315}x^7(y^{(1)})_0 - \frac{169}{315}x^8(y^{(0)})_0 + \frac{646}{2835}x^9(y^{(1)})_0 + \dots$$

$$\therefore y = (y^{(0)})_0\left[1 - x^2 - \frac{2x^4}{3} - \frac{26x^6}{45} - \frac{169x^8}{315} + \dots\right]$$

$$+ (y^{(1)})_0\left[x + \frac{x^3}{3} + \frac{4}{15}x^5 + \frac{76}{315}x^7 + \frac{646}{2835}x^9 + \dots\right]$$

(2)(i) $L\{3e^{-4t} - 5e^{4t}\} = 3L\{e^{-4t}\} - 5L\{e^{4t}\} = \frac{3}{s+4} - \frac{5}{s-4}$ //

(ii) $L\{\sin 4t + \cos 4t\} = \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

(iii) $L\{t^3 + 2t^2 - t + 4\}$

$= L\{t^3\} + 2L\{t^2\} - L\{t\} + L\{4\}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$ //

$$L\left\{\frac{e^{-t} - e^{-2t}}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$$
$$= \left[\ln(\sigma+1) - \ln(\sigma+2)\right]_s^{\infty}$$
$$= \left[\ln\left(\frac{\sigma+1}{\sigma+2}\right)\right]_s^{\infty}$$
$$= \ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\left(\frac{s+1}{s+2}\right)$$
$$= \ln 1 - \ln\left(\frac{s+1}{s+2}\right) = 0 - \ln\left(\frac{s+1}{s+2}\right) = -\ln\left(\frac{s+1}{s+2}\right) //$$

(vii) $L\{e^{4t}\cos 2t\} = \dfrac{s-4}{(s-4)^2+4}$ //

(viii) $L\{t \sin 2t\}$

Soln

$$L\{\sin 2t\} = \frac{2}{s^2+4}$$
$$L\{t \sin 2t\} = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

Solving $\dfrac{d}{ds}\left(\dfrac{2}{s^2+4}\right)$

Using Quotient rule $\left[\dfrac{Vdu - udV}{V^2}\right]$

$u = 2 \qquad V = s^2+4$

$du = 0 \qquad dV = 2s$

$$\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{(s^2+4)0 - 2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

$$L\{t \sin 2t\} = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right) = \frac{4s}{(s^2+4)^2} //$$

(ix) $L\{t^3 + 4t^2 + 5\}$
$$= L\{t^3\} + 4L\{t^2\} + L\{5\}$$
$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s} //$$

(x) $L\{e^{3t}(t^2+4)\}$

$= L\{t^2 e^{3t}\} + L\{4e^{3t}\}$

$= \frac{2}{(s-3)^3} + \frac{4}{s-3}$ //

(xi) $L\{t^2 \cos t\}$

Soln

$L\{\cos t\} = \frac{s}{s^2+1}$

$\frac{-d}{ds}\left(\frac{s}{s^2+1}\right) = -\left(\frac{(s^2+1)1 - s(2s)}{(s^2+1)^2}\right) = -\left(\frac{s^2+1-2s^2}{(s^2+1)^2}\right) = -\left(\frac{1-s^2}{(s^2+1)^2}\right) = \frac{s^2-1}{(s^2+1)^2}$

$L\{t^2 \cos t\} = -\frac{d}{ds}\left(-\frac{d}{ds}\left(\frac{s}{s^2+1}\right)\right) = -\frac{d}{ds}\left(\frac{s^2-1}{(s^2+1)^2}\right)$

$\frac{-d}{ds}\left(\frac{s^2-1}{(s^2+1)^2}\right) = -\left(\frac{(s^2+1)^2 \times 2s - (s^2-1)\cdot 4s(s^2+1)}{(s^2+1)^4}\right)$

$= -\left[\frac{2s(s^2+1)^2 - 4s(s^2-1)(s^2+1)}{(s^2+1)^4}\right]$

$= -\frac{\cancel{(s^2+1)}\left[2s(s^2+1) - 4s(s^2-1)\right]}{(s^2+1)^{\cancel{4}\,3}}$

$= \frac{-2s(s^2+1-2s^2+2)}{(s^2+1)^3}$

$= \frac{2s(-s^2-1+2s^2-2)}{(s^2+1)^3}$ //

$u = s^2-1 \qquad v = (s^2+1)^2$

$du = 2s \qquad dv = 4s(s^2+1)$

Solving for dv

$(s^2+1)^2$

$u = s^2+1 \quad ; \quad y = u^2$

$\frac{du}{ds} = 2s \qquad \frac{dy}{du} = 2u$

$\frac{dy}{ds} = \frac{dy}{du} \times \frac{du}{ds}$

$= 2u \times 2s$

$= 4su = 4s(s^2+1)$

(xii) $L\left\{\frac{\sinh 2t}{t}\right\}$

Soln

$$\lim_{t \Rightarrow 0}\left\{\frac{\sinh 2t}{t}\right\} = \frac{\sinh 2(0)}{0} = \frac{0}{0}$$

Using L'Hopital's rule

$$\lim_{t \Rightarrow 0}\left\{\frac{2\cosh 2t}{1}\right\} = \frac{2\cosh 2(0)}{1} = 2 \times 1 = 2$$

$$L\left\{\frac{\sinh 2t}{t}\right\}$$

$$L\{\sinh 2t\} = \frac{2}{s^2-4}$$

$$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{\sigma=s}^{\infty}\frac{2}{\sigma^2-4}\,d\sigma = \int_{\sigma=s}^{\infty}\frac{2}{(\sigma-2)(\sigma+2)}\,d\sigma$$

$$\frac{2}{(\sigma-2)(\sigma+2)} = \frac{A}{\sigma-2} + \frac{B}{\sigma+2} = \frac{A(\sigma+2) + B(\sigma-2)}{(\sigma-2)(\sigma+2)}$$

$$2 = A(\sigma+2) + B(\sigma-2)$$

when $\sigma = 2$

$2 = A(2+2)$

$2 = 4A$

$A = \frac{2}{4} = \frac{1}{2}$

when $\sigma = -2$

$2 = B(-2-2)$

$2 = -4B$

$B = -\frac{2}{4} = -\frac{1}{2}$

$$\therefore \int_{\sigma=s}^{\infty}\frac{2}{(\sigma-2)(\sigma+2)}\,d\sigma = \int_{\sigma=s}^{\infty}\frac{1/2}{\sigma-2}\,d\sigma - \int_{\sigma=s}^{\infty}\frac{1/2}{\sigma+2}\,d\sigma$$

$$\Rightarrow \frac{1}{2}\int_{\sigma=s}^{\infty}\frac{1}{\sigma-2}\,d\sigma - \frac{1}{2}\int_{\sigma=s}^{\infty}\frac{1}{\sigma+2}\,d\sigma$$

$$\Rightarrow \frac{1}{2}\Big[\ln(\sigma-2) - \ln(\sigma+2)\Big]_s^{\infty}$$

$$= \frac{1}{2}\left[\ln\left(\frac{\sigma-2}{\sigma+2}\right)\right]_s^{\infty}$$

$$= \frac{1}{2}\left[\ln\left(\frac{\infty-2}{\infty+2}\right) - \ln\left(\frac{s-2}{s+2}\right)\right]$$

$$= \frac{1}{2}\left[\ln 1 - \ln\left(\frac{s-2}{s+2}\right)\right]$$

$$= \frac{1}{2}\left[0 - \ln\left(\frac{s-2}{s+2}\right)\right] = -\frac{1}{2}\ln\left(\frac{s-2}{s+2}\right) //$$

(3)(i) $L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\}$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4} = \frac{A(s-4)+B(s-3)}{(s-3)(s-4)}$$

$$s-5 = A(s-4) + B(s-3)$$

when $s = 3$

$3-5 = A(3-4)$

$-2 = -A$

$A = 2$

when $s = 4$

$4-5 = B(4-3)$

$-1 = B$

$B = -1$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\} = L^{-1}\left\{\frac{2}{s-3}\right\} - L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= 2e^{3t} - e^{4t} //$$

(ii) $$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4} = \frac{A(s-4)+B(s-2)}{(s-2)(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

when $s = 2$

$2(2)-6 = A(2-4)$

$-2 = -2A$

$A = 1$

when $s = 4$

$2(4)-6 = B(4-2)$

$2 = 2B$

$B = 1$

(x $\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$$L^{-1}\left\{\frac{2s-6}{(s-2)(s-4)}\right\} = L^{-1}\left\{\frac{1}{s-2}\right\} + L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= e^{2t} + e^{4t}$$ //

(iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4} = \frac{A(s-4)+B(s)}{s(s-4)}$

$5s-8 = A(s-4) + B(s)$

when $s=0$

$5(0)-8 = A(0-4)$

$-8 = -4A$

$A = \frac{-8}{-4} = 2$

when $s=4$

$5(4)-8 = B(4)$

$12 = 4B$

$B = \frac{12}{4} = 3$

$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$

$$L^{-1}\left\{\frac{5s-8}{s(s-4)}\right\} = L^{-1}\left\{\frac{2}{s}\right\} + L^{-1}\left\{\frac{3}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$ //

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{s-1}$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)(s-1)}{(s-3)(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)(s-1)$

when $s=3$

$(3)^2 - 3(3) - 4 = A(3-1)^2$

$-4 = 4A$

$A = -1$

$B = C = 0$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3}$

$$L^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = L^{-1}\left\{\frac{-1}{s-3}\right\}$$

$$= -e^{3t} \,//$$

(v) $\dfrac{s-5}{s^2+4s+20}$

Soln

$$\frac{s-5}{s^2+4s+20} = \frac{(s+2)-7}{(s+2)^2+16}$$

$$L^{-1}\left\{\frac{s-5}{s^2+4s+20}\right\} = L^{-1}\left\{\frac{(s+2)-7}{(s+2)^2+16}\right\}$$

$$= L^{-1}\left\{\frac{(s+2)}{(s+2)^2+16}\right\} - 7L^{-1}\left\{\frac{1}{(s+2)^2+16}\right\}$$

$$= e^{-2t}\cos 4t - 7e^{-2t}\sin 4t \,//$$