$$= \frac{s+2}{(s+2)^2+16} - \frac{2}{(s+2)^2+16} \cdot \frac{s}{s+2}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{(s+2)^2+4^2} \cdot -\frac{1}{4}$$

$$\Rightarrow \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$f(t) = e^{-2t}\cos 4t - \frac{7}{4} e^{-2t}\sin 4t$$

$$= e^{-2t}\left[\cos 4t - \frac{7}{4}\sin 4t\right]$$

$A = \frac{s^2-3s-4}{(s-1)^2}\Big|_{s=3} = \frac{3^2-3(3)-4}{(3-1)^2} = -1$

$B = \frac{s^2-3s-4}{s-3}\Big|_{s=1} = \frac{1^2-3(1)-4}{1-3} = 3$

$C = \frac{d}{ds}\left[\frac{s^2-3s-4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3)-(s^2-3s-4)}{(s-3)^2}$

at $s=1$

$\frac{(1-3)(2(1)-3)-[1^2-3(1)-4]}{(1-3)^2} = 2$

$f\,s = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$

$= -e^{3t} + 3te^{t} + 2e^{t}$

$= e^{t}(3t+2) - e^{3t}$

(v) $= \frac{s-5}{s^2+4s+20} = F(s)$

$F(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$

[illegible]

$F(s) = \frac{1}{s-2} + \frac{1}{s-4}$

$f(t) = e^{2t} + e^{4t}$

(vii) $\frac{5s-8}{s(s-4)} = F(s) = \frac{A}{s} + \frac{B}{s-4}$

$A = \frac{5s-8}{s-4}\Big|_{s=0} = \frac{5(0)-8}{0-4} = \frac{-8}{-4} = 2$

$B = \frac{5s-8}{s}\Big|_{s=4} = \frac{5(4)-8}{4} = 3$

$F(s) = \frac{2}{s} + \frac{3}{s-4}$

$f(t) = 2 + 3e^{4t}$

(ix) $\frac{s^2-3s+4}{(s-3)(s-1)^2} = F(s)$

$= \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$A = \frac{s^2 ...}{(s-...}$

$B = \frac{s^2 ...}{s-...}$

$C = \frac{d}{ds}[...$

(x) $\frac{3s...}{s^2+4s...}$

$F(s) = \frac{3...}{(s^2...}$

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$$\frac{(s^2+4)\,0 - 2(2s)}{(s^2+4)^2} = \left(\frac{-4s}{(s^2+4)^2}\right) - \Rightarrow \frac{4s}{(s^2+4)^2}$$

ix $t^3 + 4t^2 + 5 = f(t)$

$$= \frac{3!}{s^4} + \frac{4 \cdot 2!}{s^3} + \frac{5}{s} = \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

$$= \frac{1}{s^4}\left[6 + 8s + 5s^3\right.$$

xi. $t^2 \cos t = f(t)$

$$L[\cos t] = \frac{s}{s^2+1} \cdot L[t^2 \cos t] = -\frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\frac{d}{ds}\left[\frac{s}{s^2+1}\right] = \frac{(s^2+1)\cdot 1 - s(2s)}{(s^2+1)^2} = \frac{s^2+1-2s^2}{(s^2+1)^2} = \frac{1-s^2}{(s^2+1)^2}$$

$$\frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right] \therefore \begin{matrix} u = 1-s^2 & du = -2s \\ v = (s^2+1)^2 & dv = 4s(s^2+1) \end{matrix}$$

$v = s^2+1 \quad dv = 2s$

$w = v^2 \quad dw = 2v$

$$\frac{dv}{ds} \times \frac{dw}{dv} = 2s \times 2v \Rightarrow 4sv = 4s(s^2+1)$$

$$\frac{[s^2+1]^2 \cdot 2s - (1-s^2)\,4s(s^2+1)}{(s^2+1)^4} = \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)^4}$$

$$= \frac{(s^2+1)\left[-2s(s^2+1) - 4s(1-s^2)\right]}{(s^2+1)^{4\,3}} = \frac{-2s\left[s^2+1-2+2s^2\right]}{(s^2+1)^3}$$

$$= \frac{-2s\left[3s^2-1\right]}{(s^2+1)^3} \quad \because L[t^2\cos t] = \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$$

$$\therefore \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right] = \frac{2s\left[3s^2-1\right]}{(s^2+1)^3}$$

(xii) $\frac{\sinh 3t}{t} = f(t)$

$$\lim_{t \to 0}\left[\frac{\sinh 2t}{t}\right] = \frac{2\cosh 2t}{1} = \frac{2}{1} = 2$$

$$L\left[\frac{\sinh 2t}{t}\right] = L[\sinh 2t] = \frac{2}{s^2-2^2} = \frac{2}{s^2-4}$$

$$L\left[\frac{\sinh 2t}{t}\right] = \int_{\sigma=s}^{\infty} \frac{2}{\sigma^2-4}\, d\sigma = 2\int_{\sigma=s}^{\infty} \frac{1}{\sigma^2-4}\, d\sigma$$

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(i) $\frac{s-5}{(s-3)(s-4)} = f(s) = \frac{A}{(s-3)} + \frac{B}{(s-4)}$

A: $s-3 \cdot f(s) = \frac{(s-3) \cdot (s-5)}{(s-3)(s-4)}\Big|_{s=3} = \frac{(3-5)}{(3-4)} = 2$

B: $s-4 \cdot f(s) = \frac{(s-4) \cdot (s-5)}{(s-3)(s-4)}\Big|_{s=4} = \frac{4-5}{4-3} = -1$

$f(s) = \frac{2}{s-3} - \frac{1}{s-4}$

$f(t) = 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = f(s) = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$A = \frac{2s-6}{s-4}\Big|_{s=2} = \frac{2(2)-6}{2-4} = 1$

$B = \frac{2s-6}{s-2}\Big|_{s=4} = \frac{2(4)-6}{4-2} = 1$

$y = (y^0)_0 + x(y^1)_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(y^{(3)})_0 + \ldots$

$y = (y^0)_0 + x(y^{(1)})_0 + \frac{x^2}{2!}(y^{(2)})_0 + \frac{x^3}{3!}(0)_0 + \frac{x^4}{4!}(4)(-2)(y^{(1)})_0$

$\ldots + \frac{x^5}{5!}(0) + \frac{x^6}{6!}(5)(4)(-2)(y^{(2)})_0 + \frac{x^7}{7!}(0)$

$y = (y^0)_0 + x(y^1)_0 - x(y^0)_0 - \frac{x^4}{3 \times 1}(y^{(1)})_0 - \frac{x^6}{3}(y^{(3)})_0$

$y = (y^0)_0 [1 + x - \frac{x^2}{3} - \frac{x^6}{6}] + (y^{(1)})_0 [x \ldots]$

② (i) $3e^{-4t} - 5e^{4t} = f(t)$

$\mathcal{L}[f(t)] = \frac{3}{s+4} - \frac{5}{s-4} = \frac{3(s-4) - 5(s+4)}{(s+4)(s-4)} \Rightarrow \frac{3s - 12 - 5s - 20}{(s+4)(s-4)}$

(ii) $\sin 4t + \cos 4t = f(t)$

$\mathcal{L}[f(t)] = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2} = \frac{4+s}{(s^2+4^2)}$

(iii) $t^3 + 2t^2 - 6t + 4$

$\frac{n!}{s^{n+1}} = \frac{3!}{s^4} + \frac{2 \times 2!}{s^3} - \frac{6 \cdot 1!}{s^2} + \frac{4}{s} = \frac{6}{s^4} + \frac{4}{s^3} - \frac{6}{s^2} + \frac{4}{s}$

$= \frac{1}{s^4}\left[6 + 4s - s^2 + 4s^2\right]$

(iv) $t\sin 3t = f(t)$

$L[f(t)] = \frac{3}{s^2+3^2} = \frac{3}{s^2+9} = L[\sin 3t]$

$L[t\sin 3t] = -\frac{d}{ds} f(s)$

$u = 3 \quad v = s^2+9$

$du = 0 \quad dv = 2s$

$\frac{(s^2+9)0 - 3(2s)}{(s^2+9)^2} = -\left[\frac{-6s}{(s^2+9)^2}\right] = \frac{6s}{(s^2+9)^2}$

(v) $e^{-2t}\cos 5t = f(t)$

$L[f(t)] = L[\cos 5t] = \frac{s}{s^2+5^2}$

$L[f(t)] = \frac{s+2}{(s+2)^2+5^2}$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

$\lim_{t\to 0}\left[\frac{-e^{-t}+2e^{-2t}}{1}\right] = \frac{2}{1} = 2$

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MATH ASSIGNMENT (ENG381)

7) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y = 0$

$y^{(n)} = u^{n}v + n u^{(n-1)}v^{(1)} + \frac{n(n-1)}{2!}u^{(n-2)}v^{(2)} + \dots$

$[y^{(2+n)}\cdot(1-x^2) + ny^{(1+n)}\cdot(-2x) + \frac{n(n-1)}{2!}y^{(n)}\cdot(-2)] +$

$[y^{(1+n)}\cdot -2x + ny^{(n)}\cdot -2] + [2y^{(n)}] = 0$

$(1-x^2)y^{(2+n)} - 2xny^{(1+n)} - n(n-1)y^{(n)} - 2xy^{(1+n)} - 2ny^{(n)} + 2y^{(n)} = 0$

Let $x = 0$

$y^{(2+n)} - n(n-1)y^{(n)} - 2ny^{(n)} + 2y^{(n)} = 0$

$y^{(2+n)} + y^{(n)}[-n(n-1) - 2n + 2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2 + n - 2n + 2] = 0$

$y^{(2+n)} + y^{(n)}[-n^2 - n + 2] = 0$

$(y^{(2+n)})_0 = -(y^{(n)})_0[-n^2 - n + 2]$ ← recurrence relation

$n=0$: $(y^{(2)})_0 = -(y^{(0)})_0\cdot(2) = -2(y^{(0)})_0$

$n=1$: $(y^{(3)})_0 = -(y^{(1)})_0\cdot[0] = 0$

$n=2$: $(y^{(4)})_0 = -(y^{(2)})_0\cdot[-4] = 4(y^{(2)})_0 = (4)(-2)(y^{(0)})_0$

$n=3$: $(y^{(5)})_0 = -(y^{(3)})_0\cdot[-10] = (10)(0) = 0$

$n=4$: $(y^{(6)})_0 = -(y^{(4)})_0\cdot[-18] = (18)(4)(-2)(y^{(0)})_0$

$n=5$: $(y^{(7)})_0 = -(y^{(5)})_0\cdot[-28] = (28)(0) = 0$

$L[e^{-t} - e^{-2t}] = \frac{1}{s+1} - \frac{1}{s+2}$

$L\left[\frac{e^{-t} - e^{-2t}}{t}\right] = \int_{\sigma=s}^{\infty} \left(\frac{1}{\sigma+1}\right) - \left(\frac{1}{\sigma+2}\right) d\sigma$

$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} d\sigma - \int_{\sigma=s}^{\infty} \left(\left(\frac{1}{\sigma+1}\right)\right) d\sigma$

$= [\ln(\sigma+1) - \ln(\sigma+2)]_{\sigma}^{\infty}$

$= [\ln(\sigma+1) - \ln(\sigma+2)]_{s}^{\infty}$

$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_{s}^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right]$

$= -\ln\left[\frac{s+1}{s+2}\right] = \ln\left[\frac{s+2}{s+1}\right]$

(vii) $e^{2t}\cos 2t$

$L(\cos 2t) = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$

$L[e^{2t}\cos 2t] = \frac{s-2}{(s-2)^2+4}$

(viii) $t\sin 2t$

$L(\sin 2t) = \frac{2}{s^2+2^2}$

$L(t\sin 2t) = -\frac{d}{ds}[f(s)]$

$u = 2 \quad du = 0 \; ; \; v = s^2+4 \quad dv = 2s$