ASSIGNMENT 1

(1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$m^2 - m - 2 = 0$

$m(m+1) - 2(m+1) = 0$

$m = 2, \quad m = -1$

$C.F \Rightarrow y = Ae^{2x} + Be^{-x}$

$y = 4$

$\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

$2x = 0$

$x = 0$

$y = 4$

$G.S \Rightarrow C.F + P.I$

$= Ae^{2x} + Be^{-x} = 4$

(2) $\frac{d^2y}{dx^2} - 4y = 10e^{5x}$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm\sqrt{4} = \pm 2$

$C.F \Rightarrow y = C\cosh 2x + D\sinh 2x$

$y = Ce^{2x}$

$\frac{dy}{dx} = 2Ce^{2x}$

15/03/20

$d^2y/dx^2 = 4ce^{2x}$

$9e^{2x} - 4ce^{2x} = 10e^{2x}$

$5e^{2x} = 10e^{2x}$

$5c = 10$

$c = 10/5 = 2$

PI $\Rightarrow y = 2e^{2x}$

GS = $C\cosh 2x + D\sinh 2x + 2e^{2x}$

③ $d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

$m^2 + 2m + 1 = 0$

$m(m+1) + 1(m+1) = 0$

$m = m_1 = -1$

CF $\Rightarrow y = e^{-x}(A + Bx)$

$y = Ce^{-2x}$, $dy/dx = -2Ce^{-2x}$, $d^2y/dx^2 = 4Ce^{-2x}$

$d^2y/dx^2 + 2dy/dx + y = e^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$C = 1$

PI $y = e^{-2x}$ } GS = $e^{-x}(A + Bx) + e^{-2x}$

④ $d^2y/dx^2 + 25y = 5x^2 + x$

$m^2 + 25 = 0$

$m = \pm\sqrt{-25}$

$m = \pm j5$

CF $y = C\cos 5x + D\sin 5x$

$y = Cx^2 + Dx + E$

$dy/dx = 2Cx + D$

$d^2y/dx^2 = 2C$

$d^2y/dx^2 + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$2C + 25E + 25Cx^2 + 25Dx = 5x^2 + x$

15/[illegible]/07

$2C + 25E = 0$ ——— ①

$25C = 5$ ——— ②

$C = \frac{1}{5}$

$25D = 1$ ——— ③

$D = \frac{1}{25}$

$2\left(\frac{1}{5}\right) + 25E = 0$

$E = \frac{-2}{125}$

$GS = C\cos 5x + D\sin 5x + \frac{1}{5}x + \frac{1}{25}x^2 - \frac{2}{125}$

⑥ $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m_1 = 1, m_2 = 1$

$\Rightarrow y = e^x(A + Bx)$

$y = C\cos x + D\sin x$

$y' = -C\sin x + D\cos x$

$y'' = -C\cos x - D\sin x$

$-C\cos x - D\sin x + (C\sin x + D\cos x) - (C\cos x + D\sin x) = 4\sin x$

$2C\sin x + 2D\cos x = 4\sin x$

$2C\sin x = 4\sin x$

$C = 2$

$2D\cos x = 0$

$D = 0$

then $y = 2\cos x$

$\therefore GS = e^x(A + Bx) + 2\cos x$

⑦ $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

where $x = 0, y = 1$ and $\frac{dy}{dx} = -2$

$m^2 + 4m + 5 = 0$

[illegible] $= \sqrt{(-2)}$ [illegible]

[illegible]

$= m = -2 \pm i$

$C.F = y = e^{-2x}(A\cos x + B\sin x)$

$y = Ce^{-2x}$

$y' = -2Ce^{-2x}$

$y'' = 4Ce^{-2x}$

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$4Ce^{-2x} - 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$Ce^{-2x} = 2e^{-2x}$

$C = 2$

$P.I = y = 2e^{-2x}$

$G.S = y = e^{-2x}(A\cos x - B\sin x) - 2e^{-2x}$

when $x = 0$, $y = 1$

$1 = e^{-2(0)}(A\cos(0) - B\sin(0)) - 2e^{-2(0)}$

$1 = A - 2$

$A = 1 - 2$

$A = -1$

find $\frac{dy}{dx}$, $\frac{dy}{dx} = -2$

$\frac{dy}{dx} = -2Ae^{-2x}\sin x - 2Be^{-2x}\cos x - 4e^{-2x}$

$-2 = -2Ae^{-2(0)}\sin(0) - 2Be^{-2(0)}\cos(0) - 4e^{-2(0)}$

$-2 = -2B - 4$

$-2B = 2$

$B = -1$

$y = e^{-2x}(-\cos x - B\sin x) + 2e^{-2x}$

$= -e^{-2x}(\cos x + \sin x) + 2e^{-2x}$

13/ENG03/019

(5) $3\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 2x - 3$

$3m^2 - 2m - 1 = 0$

$3m(m-1) + 1(m-1) = 0$

$(3m+1)(m-1) = 0$

$m_1 = -1/3, \; m_2 = 1$

$y = Ae^{-1/3x} + Be^{x}$

$y = Cx + D$, $y' = C$, $y'' = 0$

$3(0) - 2C - (Cx + D) = 2x - 3$

$-2C - (x - D) = 2x - 3$

$D = 7$

$P.I \Rightarrow y = -2x + 7$

$GE \Rightarrow y = Ae^{x} + Be^{-0.33x} - 2x + 7$

(4) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$m^2 - 6m + 8 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 32}}{2}$

$m_1 = 4, \; m_2 = 2 \Rightarrow y = Ae^{4x} + Be^{2x}$

$y = Cxe^{4x}, \; y' = 4Cxe^{4x} + Ce^{4x}, \; y'' = 16Cxe^{4x} + 4Ce^{4x} + 4Ce^{4x}$

$16Cxe^{4x} + 4Ce^{4x} - 6(4Cxe^{4x} + Ce^{4x}) + 8(Cxe^{4x}) = 8e^{4x}$

$4Ce^{4x} + 4Ce^{4x} - 6Ce^{4x} = 8e^{4x}$

$C(2) = 8$

$C = 4$

$y = 4xe^{4x}$

$GE = y = Ae^{4x} + Be^{2x} + 4xe^{4x}$