15/ENG03/017

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ASSIGNMENT 5

① $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=2$

$$L\left\{\frac{dy}{dt}\right\} = sy(s) - y(0)$$

$$L\{3y\} = 3y(s)$$

$$L\{e^{-2t}\} = \frac{1}{s+2}$$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1}{s+2} + 2$$

$$y(s) = \frac{1+2(s+2)}{s+2} = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

$A+B=2$ —— ① $\Rightarrow B=1$

$3A+2B=5$ —— ② $\Rightarrow A=1$

$$\frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1} = e^{-2t} + e^{-3t}$$

② $3\frac{dy}{dt} - 6y = \sin 2t$ $\quad t=0 \quad y=1$

$$3sy(s) - 6y(s) - 3 = \frac{2}{s^2+2^2}$$

$$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$$

$$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$$

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$2+3(s+2)^2 = A(s+2)(3s-6)+B(3s-6)+C(s+2)^2$

$2+3s^2+12s+12 = 3As^2-12A+3Bs-6B+Cs^2+4Cs+4C$

$3A+C=3$ ——— (1)

$3B+4C=12$ ——— (2)

$-12A-6B+4C=14$ ——— (3)

$3A = 3-C$

$A = \frac{3-C}{3}$

$3B+4AC=12$

$-12\left(\frac{3-C}{3}\right)-6B+4C=14$

$-12+4C-6B+4C=14$

$-6B+8C=28$ ——— (4)

$-18B-24C=-72$

$-18B+24C=84$

$-48C=-156$

$C = 13/4$

$3B = 12-13$

$B = -1/3$

$A = \frac{3-13/4}{3} = \frac{-1}{12}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}$

$L^{-1}\{Y(s)\} = -1/12\,e^{-2t} - 1/3\,te^{-2t} - 13e^{2t}$

$y = -1/12\,(e^{-2t}+4te^{-2t}-13e^{2t})$

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① $\frac{dy}{dt} - 4y = e^{2t}$

$sY(s) - y(0) - 4Y(s) = \frac{1}{s-2}$

$Y(s)(s-4) - 2 = \frac{1}{s-2}$

$Y(s) = \frac{s+25}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$s+25 = A(s-4) + Bs = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = -2$

$B = 2+2 = 4$

$\frac{-2}{s} + \frac{4}{s-4}$

$L^{-1}\{Y(s)\} = -2 + 4e^{4t}$

② $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ at $t=0, y=2, y'=1$

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$

$L\left\{-2\frac{dy}{dt}\right\} = -2sY(s) + 2y(0)$

$L\{5y\} = 5Y(s)$

$L\{e^{2t}\} = \frac{1}{s-2}$

$s^2Y(s) - 5y(0) - y'(0) - 2sY(s) + 2y(0) + 5Y(s) = \frac{1}{s-2}$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2 - 1 + 4 = \frac{1}{s-2}$

$Y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$Y(s) = \frac{1 + 2s^2 - 7s + 6}{(s-2)(s^2-2s+5)} = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$

$= \frac{A}{(s-2)} + \frac{Bs + C}{(s^2-2s+5)}$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

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$A+B=2$ ——— ①

$-2A-2B+C=-7$ ——— ②

$5A-2C=7$ ——— ③

$B=2-A$

$-2A-2(2-A)+C=-7$

$C=-3$

$5A-2(-3)=7$

$A=1/5$

$A+B=2$

$\frac{1}{5}-B=2$

$B=\frac{9}{5}$

$\frac{2s^2-2s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5 s^{-2}}{(s^2-2s-5)}$

$\frac{2s^2-3s-7}{(s-2)(s^2-2s+5)} = \frac{1/5}{s-2} + \frac{4/5 s}{(s^2-2s-5)} - \frac{7}{(s^2-2s+5)}$

$= \frac{1/5}{(s-2)} + \frac{4}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{7 \cdot 1/5}{(s^2-2s+5)}$

$= \frac{1}{5}\frac{1}{(s-2)} + \frac{4}{5}\left(\frac{s-1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$

$L^{-1}\{Y(s)\} = \frac{1}{5}e^{2t} + \frac{4}{5}\{e^t\cos 2t\} - \frac{3}{2}(e^t \sin 2t)$

⑤ $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = e^{3t}$ $\quad t=0,\ y=0,\ y'=2$

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2Y(s) - sy(0) - y'(0)$

$L\left\{-6\frac{dy}{dt}\right\} = -6sY(s) + 6y(0)$

$L\{9y\} = 9Y(s)$

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$$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}$$

$$s^2Y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8Y(s) = \frac{1}{s-3}$$

$$s^2Y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$Y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$Y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{(s-3)}$$

$$Y(s) = \frac{2s - 5}{(s-3)(s^2 - 6s + 8)} = \frac{A}{(s-3)} + \frac{Bs + c}{s^2 - 6s + 8}$$

$$2s - 5 = A(s^2 - 6s + 8) + (Bs + c)(s - 3)$$

$$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + cs - 3c$$

$$A + B = 0 \quad \text{——— (1)}$$

$$-6A - 3B + c = 2 \quad \text{——— (2)}$$

$$8A - 3c = -5 \quad \text{——— (3)}$$

$$B = -A$$

$$-6A + 3A + c = 2$$

$$-3A + c = 2 \quad \text{— (4)}$$

$$8A - 3c = -5 \quad \text{— (5)}$$

$$9A - 3c = 6$$

$$8A - 3c = -5$$

$$A = -1$$

$$B = 1$$

$$\frac{2s - 5}{(s-3)(s^2 - 6s + 8)} = \frac{-1}{(s-3)} + \frac{s - 1}{(s^2 - 6s + 8)}$$

$$\frac{2s - 5}{(s-3)(s^2 - 6s + 8)} = \frac{-1}{(s-3)} + \frac{s - 1}{(s-2)(s-4)}$$

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$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$s-1 = A(s-4) + B(s-2)$$

$A+B=1$ ——— (1)

$-4A-2B=-1$ ——— (2)

$-2B=-3$

$B = 3/2$, $A = -1/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}$$

$$\mathcal{L}^{-1}\{Y_s\} = 4 - e^{3t} - \tfrac{1}{2}e^{2t} + \tfrac{3}{2}e^{4t}$$

$$= \tfrac{1}{2}(2e^{4} + e^{2t} - 3e^{4t})$$