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Assignment iv

1) $(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' + 2xy' + 2y = 0$

for submodule 1, $w = (1-x^2)y''$

$u = y''$, $v = 1-x^2$

$u^n = y^{n+2}$, $v' = -2x$, $v'' = -2$, $v''' = 0$

$y = vu^n + nu^{n-1}v' + \frac{n(n-1)u^{n-2}v''}{2!} + \frac{n(n-1)(n-2)u^{n-3}v'''}{3!} + \dots + (n-1)(n-r+1) + v^n$

$w^n = y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)y^n(\cancel{-2})}{2!}$

$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n$

for sub 2; $w = 2xy'$

$u = y'$, $u^n = y^{n+1}$, $v = 2x$, $v' = 2$; $2x \cdot y^{n+1} + n \cdot y^n \cdot 2 = w^n$

for sub 3, $w = 2y$

$u = y$, $v = 2$

$u^n = y^n$, $v' = 0$

$w^n = y^n \cdot 2 + 0$

combining

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n + 2xy^{n+1} + 2ny^n + 2y^n = 0$

setting $x = 0$

$y^{n+2} - (n^2-n)y^n + 2ny^n + 2y^n = 0$

$y^{n+2} = (n^2-n)y^n - 2ny^n - 2y^n$

If $n = 0$, $(y^2)_0 = -2(y^0)_0$

If $n = 1$, $(y^3)_0 = -2(y^1)_0 - 2(y^1)_0$

$= -4(y^1)_0$

If $n = 2$; $(y^4)_0 = 2(y^2)_0 - 4(y^2)_0 - 2(y^2)_0$

$= -4(y^2)_0 = -4(-2(y^0)_0)$

$= 8(y^0)_0$

$y^{n+2} = (n^2-n)y^n - 2ny^n - 2y^n$

If $n=3$; $(y^5)_0 = 6(y^3)_0 - 6(y^3)_0 - 2(y^3)_0$

$= -2(y^3)_0 = -2(-4(y')_0)$

$= 8(y')_0$

If $n=4$; $(y^6)_0 = 12(y^4)_0 - 8(y^4)_0 - 2(y^4)_0$

$= 2(y^4)_0$

$= 2(8(y^0)_0)$

$= 16(y^0)_0$

If $n=5$; $(y^7)_0 = 20(y^5)_0 - 10(y^5)_0 - 2(y^5)_0$

$= 8(y^5)_0 = 8(8(y')_0)$

$= 64(y')_0$

Maclaurin series

$$y = (y)_0 + x(y')_0 + \frac{x^2(y'')_0}{2!} + \frac{x^3(y''')_0}{3!} + \cdots \frac{x^n y^n}{n!}$$

$$y = (y)_0 + x(y')_0 + \frac{x^2(-2(y)_0)}{2!} + \frac{x^3(-4(y')_0)}{3!} + \frac{x^4(8(y^0)_0)}{4!} + \frac{x^5(8(y')_0)}{5!}$$

$$+ \frac{x^6(16(y)_0)}{6!} + \frac{x^7(64(y')_0)}{7!} + \cdots$$

$$y = (y)_0 + x(y')_0 - x^2(y)_0 - \frac{2x^3(y')_0}{3} + \frac{x^4(y)_0}{3} + \frac{x^5(y')_0}{15} + \frac{x^6(y)_0}{45}$$

$$+ \frac{4x^7(y')_0}{315} + \cdots$$

$$y = \left[1 - x^2 + x^4/3 + x^6/45\right](y)_0 + \left[x - 2x^3/3 + x^5/15 + 4/315\, x^7\right](y')_0 + \cdots$$

2i) $3e^{-4t} - 5e^{4t}$

$L\{3e^{-4t} - 5e^{4t}\}$

$L\{3e^{-4t}\} - L\{5e^{4t}\}$

$3L\{e^{-4t}\} - 5L\{e^{4t}\}$

$$\frac{3}{s+4} - \frac{5}{s-4}$$

ii) $\sin 4t + \cos 4t$

$L\{\sin 4t + \cos 4t\} = L\{\sin 4t\} + L\{\cos 4t\}$

$$\frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$$

) $t^3+2t^2-t+4$
$L\{t^3+2t^2-t+4\} = L\{t^3\} + 2L\{t^2\} - L\{t\} + L\{4\}$
$= \frac{6}{t^4} + \frac{2\times 2}{t^3} - \frac{1}{t^2} + \frac{4}{s}$

) $e^{-2t}\cos 5t$
$L\{e^{-2t}\cos 5t\}$
$L\{\cos 5t\} = \frac{s}{s^2+25}$
~~[illegible]~~ S is replaced by a shift of $e^{-2t}$
$\therefore L\{e^{-2t}\cos 5t\} = \frac{s+2}{(s+2)^2+25}$

) $t\sin 3t$
$L\{t\sin 3t\}$
$L\{\sin 3t\} = \frac{3}{s^2+9}$
$-F'(s) = \frac{-d}{ds}\left[\frac{3}{s^2+9}\right]$
$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $u = 3$ $du = 0$
$v = s^2+9$ $dv = 2s$
$\frac{(s^2+9)(0) - 3(2s)}{(s^2+9)^2} = \frac{-6s}{(s^2+9)^2}$ $\frac{-d}{ds} = \frac{6s}{(s^2+9)^2}$

~~$\frac{e^{-t}-e^{-2t}}{s} = \frac{e^{-t}}{s} - \frac{e^{-2t}}{s}$~~ $\frac{e^{-t}-e^{-2t}}{t} = \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$
$L\{\frac{e^{-t}}{t}\} - L\{\frac{e^{-2t}}{t}\}$
$\int_s^\infty L\{e^{-t}\}d\sigma - \int_s^\infty L\{e^{-2t}\}d\sigma = \frac{1}{s+1} - \frac{1}{s+2}$
$[\ln(\sigma+1)]_s^\infty - [\ln(\sigma+2)]_s^\infty$
$= -\ln(s+1) + \ln(s+2)$
$= -\ln(s+1) + \ln(s+2)$
$= -\ln\left[\frac{(s+2)}{(s+1)}\right]$

vii) $e^{4t}\cos 2t$

$L\{e^{4t}\cos 2t\}$

$L\{\cos 2t\} = \dfrac{s}{s^2+4}$

s is replaced by a shift of $e^{4t}$, s is replaced by s-4

$$\frac{s-4}{(s-4)^2+4}$$

viii) $t\sin 2t$

$L\{\sin 2t\} = \dfrac{2}{s^2+4}$

$L\{t\sin 2t\} = -\dfrac{d}{ds}\left[\dfrac{2}{s^2+4}\right]$

$u = 2$, $du = 0$

$v = s^2+4$, $dv = 2s$

$$\frac{(s^2+4)(0) - 2(2s)}{(s^2+4)^2} = \frac{-4s}{(s^2+4)^2}$$

ix $t^3+4t^2+5$

$L\{t^3+4t^2+5\} = L\{t^3\} + L\{4t^2\} + L\{5\}$

$$\frac{6}{t^4} + 4\left(\frac{2}{t^3}\right) + \frac{5}{s}$$

$$6/s^4 + 8/s^3 + 5/s$$

x) $e^{3t}(t^2+4)$

$L\{t^2+4\} = L\{t^2\} + L\{4\}$

$$= \frac{2}{s^3} + \frac{4}{s}$$

s is replaced by a shift of $e^{3t}$

$$\frac{2}{s^3} - \frac{4}{s-3}$$

xi) $t^2\cos t$ $L\{t^2\cos t\}$

$L\{\cos t\} = \dfrac{s}{s^2+1}$

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2 xi) $L\{\cos t\} = \frac{s}{s^2+2}$

$L\{t^2 \cos t\} = f''(s) = -\frac{d^2}{ds^2}\left\{\frac{s}{s^2+2}\right\}$

$-f'(s) = -\frac{d}{ds}\left\{\frac{s}{s^2+2}\right\}$

$u = s \quad , v = s^2+2$

$du = 1 \quad dv = 2s$

$\frac{(s^2+2)(1) - (s)(2s)}{(s^2+2)^2} = \frac{s^2+2-2s^2}{(s^2+2)^2} = \frac{-s^2+2}{(s^2+2)^2}$

$= \frac{s^2+2}{(s^2+2)^2}$

$-f'(s) = -\frac{d}{ds}\left\{\frac{s^2+2}{(s^2+2)^2}\right\}$

$u = s^2+2 \quad v = (s^2+2)^2$

$du = 2s \quad dv = 4s(s^2+2) = 4s^3+4s$

$\frac{(s^2+2)^2(2s) - (s^2+2)(4s^3+4s)}{(s^2+2)^4} = \frac{(s+2)(2s) - (4s^3+4s)}{(s^2+2)^3}$

$\frac{2s^2+4s-4s^3-4s}{(s^2+2)^3} = \frac{2s^2-4s^3}{(s^2+2)^3} = \frac{2s^2[1-2s]}{(s^2+2)^3}$

$= \frac{-2s^2[1-2s]}{(s^2+2)^3}$

xii) $\frac{\sinh at}{t}$

$L\{\sinh at\} = \frac{s}{s^2-4}$

$f\sigma = \frac{s}{\sigma^2-4}$

$L\left\{\frac{\sinh at}{t}\right\} = \int_{\sigma=s}^{\infty} \frac{s}{\sigma^2-4}\, d\sigma = \left(\tan^{-1}\frac{\sigma}{2}\right]_s^{\infty}$

$= [\tan^{-1}\infty/2 - \tan^{-1} s/2]$

$= 90 - \tan^{-1} s/2 = \tan^{-1} 2/s$

3)

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$$

$$s-5 = A(s-4) + B(s-3)$$

at $s=4$; $-1 = B$ , $B=-1$

at $s=3$; $-2 = -A$

$$A = 2$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{2}{s-3}\right\} - L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$2\left[e^{3t}\right] - e^{4t}$$

$$= 2e^{3t} - e^{4t}$$

ii)

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$2s-6 = A(s-4) + B(s-2)$$

at $s=2$; $-2 = -2A$

$$A = 1$$

at $s=4$; $2 = 2B$

$$B = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left\{\frac{1}{s-2}\right\} + L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= e^{2t} + e^{4t}$$

iii)

$$\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$5s-8 = A(s-4) + B(s)$$

at $s=0$; $-8 = -4A$

$$A = 2$$

at $s=4$; $12 = 4B$

$$B = 3$$

$$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$$

$$L^{-1}\left\{\frac{2}{s}\right\} + L^{-1}\left\{\frac{3}{s-4}\right\} = 2L^{-1}\left\{\frac{1}{s}\right\} + 3L^{-1}\left\{\frac{1}{s-4}\right\}$$

$$= 2 + 3e^{4t}$$

iv) $$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$$

$$s^2-3s-4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$$

If $s=1$; $-6 = -2C$

$C = 3$

If $s=3$; $-4 = 4A$

$A = -1$

If $s=0$; $-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3(3)$

$-4+1+9 = 3B$

$6 = 3B$, $B = 2$

$$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$$

$$L^{-1}\left\{\frac{-1}{s-3}\right\} + 2L^{-1}\left\{\frac{1}{s-1}\right\} + 3L^{-1}\left\{\frac{1}{(s-1)^2}\right\}$$

$$-e^{3t} + 2e^{t} + 3te^{t}$$

v) $$\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+16+4} = \frac{s-5}{s^2+4s+20} = \frac{s-5}{s(s+4)+20} = \frac{A}{s+20} + \frac{B}{s+4+20}$$

$s-5 = A(s+24) + B(s+20)$

If $s=-20$; $-25 = 4A$, $A = -25/4$

If $s=-24$; $-29 = $

$1 = $

$$\frac{s-5}{(s+2)(s+2)+16} = \frac{s-5}{(s+2)^2+4^2} = \frac{s}{(s+2)^2+4^2} - \frac{5}{4\left((s+2)^2+4^2\right)}$$

$$L^{-1}\left\{\frac{5}{4(s+2)^2+4^2}\right\} = e^{-2t}\cos 4t - \frac{5}{4}e^{-2t}\sin 4t$$