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13/ENG06/059
Mechanical Engineering.
ENG 381
Assignment

$$(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2y = 0$$

$$(1-x^2)y'' + 2xy' + 2y = 0$$

for submodule 1, $w = (1-x^2)y''$

$u = y''$ $\qquad v = 1-x^2$

$u^n = y^{n+2}$ $\qquad v' = -2x$, $v'' = -2$, $v''' = 0$

$$y = vu^n + nu^{n-1}v' + \frac{n(n-1)}{2!}u^{n-2}v''$$

$$w^n = y^{n+2}(1-x^2) + ny^{n+1}(-2x) + \frac{n(n-1)}{2!}y^n(-2)$$

$$= (1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n$$

for submodule 2 $\quad w = 2xy'$

$u = y'$, $u^n = y^{n+1}$, $v = 2x$, $v' = 2$; $2x\,y^{n+1} + ny^n \cdot 2 = w^n$

for sub 3, $w = 2y$

$u = y$ $\qquad v = 2$

$u^n = y^n$ $\qquad v' = 0$

$w^n = y^n \cdot 2 + 0$

Combining sub 1 + sub 2 + sub 3

$$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n + 2xy^{n+1} + 2ny^n + 2y^n$$

setting $x = 0$

$$y^{n+2} - (n^2-n)y^n + 2ny^n + 2y^n = 0$$

$$y^{n+2} = (n^2-n)y^n - 2ny^n - 2y^n$$

if $n = 0$; $\quad y''_{(0)} = -2y_{(0)}$

if $n = 1$; $\quad y'''_{(0)} = -2y'_{(0)} - 2y'_{(0)}$

$$= -4y'_{(0)}$$

if $n = 2$; $\quad y^{iv}_{(0)} = 2y''_{(0)} - 4y''_{(0)} - 2y''_{(0)}$

$= -4y^{2}_{(0)} = -4(-2(y^{0}_{(0)})$

$= 8y^{\prime}_{(0)}$

$y^{n+2} = (n^2 - n)y^n - 2ny^n - 2y^n$

if $n = 3$; $y^{5}_{(0)} = 6y^{3}_{(0)} - 6y^{3}_{(0)} - 2y^{3}_{(0)}$

$= -2y^{3}_{(0)} = -2(-4(y^{\prime}_{(0)}))$

$= 8y^{\prime}_{(0)}$

if $n = 4$: $y^{6}_{(0)} = 12y^{4}_{(0)} - 8y^{4}_{(0)} - 2y^{4}_{(0)}$

$= 2y^{4}_{(0)}$

$= 2(8y^{0}_{(0)})$

$= 16y^{0}_{(0)}$

if $n = 5$; $y^{7}_{(0)} = 20y^{5}_{(0)} - 10y^{5}_{(0)} - 2y^{5}_{(0)}$

$= 8y^{5}_{(0)} = 8(8y^{\prime}_{(0)}) = 64y^{\prime}_{(0)}$

maclaurin series.

$y = y_{(0)} + xy^{\prime}_{(0)} + \frac{x^2 y^{\prime\prime}_{(0)}}{2!} + \frac{x^3 y^{\prime\prime\prime}_{(0)}}{3!} + \cdots \frac{x^n y^n}{n!}$

$y = y_{(0)} + xy^{\prime}_{(0)} + \frac{x^2}{2!}(-2y_{(0)}) + \frac{x^3}{3!}(-4y^{\prime}_{(0)}) + \frac{x^4}{4!}(8y^{0}_{(0)}) + \frac{x^5}{5!}(8y^{\prime}_{(0)})$

$+ \frac{x^6}{6!}(16y^{0}_{(0)}) + \frac{x^7}{7!}(64y^{\prime}_{(0)}) + \cdots$

$y = y_{(0)} + xy^{\prime}_{(0)} - x^2 y_{(0)} - \frac{2x^3}{3}y^{\prime}_{(0)} + \frac{x^4}{3}y^{0}_{(0)} + \frac{x^5}{15}y^{\prime}_{(0)} + \frac{x^6}{45}y_{(0)}$

$+ \frac{4x^7}{315}(y^{\prime})$.

$y = (1 - x^2 + x^4/3 + x^6/45)y_{(0)} + [x - \frac{2x^3}{3} + \frac{x^5}{15} + \frac{4}{315}x^7]y^{\prime}_{(0)} + \cdots$

$= 2e^{-4t} - 5e^{4t}$

$L\{2e^{-4t} - 5e^{4t}\}$

$\mathcal{L}\{3e^{-4t}\} - \mathcal{L}\{5e^{4t}\}$

$3\mathcal{L}\{e^{-4t}\} - 5\mathcal{L}\{e^{4t}\}$

$\frac{3}{s+4} - \frac{5}{s-4}$

(ii) $\sin 4t + \cos 4t$

$\mathcal{L}\sin 4t + \mathcal{L}\cos 4t$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{s^2+16}$

(iii) $t^3 + 2t^2 - t + 4$

$\mathcal{L}t^3 + \mathcal{L}2t^2 - \mathcal{L}t + \mathcal{L}4$

$\frac{6}{s^4} + \frac{2\times 2}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t}\cos 5t$.

$\mathcal{L}\{e^{-2t}\cos 5t\}$

$\cos 5t = \frac{5}{s^2+25}$

s is replaced by a shift of $e^{-2t} = (s+2)$

$= \frac{5}{(s+2)^2+25}$

v) $t\sin 3t$

$\mathcal{L}\{t\sin 3t\}$

$\sin 3t = \frac{3}{s^2+9}$ $-u$ $-v$

$-f'(s) = \frac{-(s^2+9)(0) + 3(2s)}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$

(vi) $\frac{e^{-t} - e^{-2t}}{t} = \frac{e^{-t}}{t} - \frac{e^{-2t}}{t}$

$\mathcal{L}\{\frac{e^{-t}}{t}\} - \mathcal{L}\{\frac{e^{-2t}}{t}\}$

$\mathcal{L}\{e^{-t}\} - \mathcal{L}\{e^{-2t}\} = \frac{1}{s+1} - \frac{1}{s+2}$

$\int_{s}^{\infty} \frac{1}{\sigma+1} d\sigma - \int_{s}^{\infty} \frac{1}{\sigma+2} d\sigma$

$= [\ln(\sigma+1)]_s^\infty - [\ln(\sigma+2)]_s^\infty$

$= -\ln(s+1) + \ln(s+2)$

$= -\ln(s+1) + \ln(s+2)$

$= -\ln\left(\frac{s+2}{s+1}\right)$

vii) $e^{4t}\cos 2t$

$\mathcal{L}\{e^{4t}\cos 2t\}$

$\mathcal{L}\cos 2t = \frac{s}{s^2+4}$

s is replaced by $(s-4)$

$\mathcal{L}\, e^{4t}\cos 2t = \frac{(s-4)}{(s-4)^2+4}$

(viii) $\mathcal{L}\, t\sin 2t$

$\mathcal{L}\{t\sin 2t\} = \frac{2}{s^2+4} \quad \frac{-4}{v}$

$\mathcal{L}(t\sin 2t) = -\frac{(s^2+4)(0) + 2(2s)}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$

ix) $t^3 + 4t^2 + 5$

$\mathcal{L}\{t^3 + 4t^2 + 5\} = \frac{6}{s^4} + 4\left\{\frac{2}{s^3}\right\} + \frac{5}{s}$

$\frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

x) $e^{3t}(t^2+4)$

$\mathcal{L}(t^2+4 = \frac{2}{s^3} + \frac{4}{s}$

replacing s by $(s-3)$

$\frac{2}{s^3} + \frac{4}{(s-3)}$

x:) $t^2 \cos t$

$\mathcal{L}\{t^2 \cos t\}$

$\mathcal{L}\{\cos t\} = \dfrac{s}{s^2+2}$

$\mathcal{L}\{t^2 \cos t\} = f''(s) = \dfrac{d^2}{ds^2}\left\{\dfrac{s}{s^2+2}\right\}$

$-f'(s) = \dfrac{-d}{ds}\left\{\dfrac{s}{s^2+2}\right\}$

$u = s \qquad v = s^2+2$

$du = 1 \qquad dv = 2s$

$\dfrac{s^2+2(1) - s(2s)}{(s^2+2)^2} = \dfrac{s^2+2-2s^2}{(s^2+2)^2} = \dfrac{-s^2+2}{(s^2+2)^2}$

$= \dfrac{s^2+2}{(s^2+2)^2}$

$-f'(s) = -\dfrac{d}{ds}\left\{\dfrac{s^2+2}{(s^2+2)^2}\right\}$

$u = s^2+2 \qquad v = (s^2+2)^2$

$du = 2s \qquad dv = 4s(s^2+2) = 4s^3+4s$

$\dfrac{(s^2+2)^2(2s) - (s^2+2)(4s^3+4s)}{(s^2+2)^4} = \dfrac{(s^2+2)(2s) - 4s^3+4s}{(s^2+2)^3}$

$\dfrac{2s+4s-4s^3-4s}{(s^2+2)^3} = \dfrac{2s^2-4s^3}{(s^2+2)^3} = \dfrac{2s^2(1-2s)}{(s^2+2)^3}$

$= \dfrac{-2s^2(1-2s)}{(s^2+2)^3}$

x(ii) $\dfrac{\sinh 2t}{t}$

$\mathcal{L}\{\sinh 2t\} = \dfrac{s}{s^2-4}$

$F(s) = \frac{s}{s^2-4}$

$L\left\{\frac{\sinh 2t}{t}\right\} = \int_{s}^{\infty} \frac{s}{s^2-4}\, ds = \left(\tan^{-1} s/2\right]_s^{\infty}$

$= \left[\tan^{-1}\frac{\infty}{2} - \tan^{-1} s/2\right]$

$= 90 - \tan^{-1} s/2 = \tan^{-1} 2/s$

2(i) $\frac{s-5}{(s-3)(s-4)} = \frac{A}{s-3} + \frac{B}{s-4}$

$s-5 = A(s-4) + B(s-3)$

At $s=4$; $B = -1$.

At $s=3$; $A = -2$

$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} - \frac{1}{s-4}$

$L^{-1}\left\{\frac{2}{s-3}\right\} - L^{-1}\left\{\frac{1}{s-4}\right\}$

$= 2e^{3t} - e^{4t}$

(ii) $\frac{2s-6}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

$-2 = -2A$

$A = 1$

at $s=2$; $a-2 = -2A$

$A = 1$

At $s=4$; $2 = 2B$

$B = 1$

$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$

$L^{-1}\frac{1}{s-2} + L^{-1} 1/s-4 = e^{2t} + e^{4t}$

x (iii) $\frac{5s-8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$5s - 8 = A(s-4) + Bs$

for $s=0$; $8 = -4A$

$A = 2$

if $s=4$; $12 = 4B$

$B = 3$

$\frac{5s-8}{s(s-4)} = \frac{2}{s} + \frac{3}{s-4}$

$\mathcal{L}^{-1}\frac{2}{s} + \mathcal{L}^{-1}\frac{3}{s-4} = 2 + 3e^{4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{(s-3)} + \frac{B}{(s-1)} + \frac{C}{(s-1)^2}$

$s^2 - 3s - 4 = A(s-1)^2 + B(s-3)(s-1) + C(s-3)$

if $s=1$; $-6 = -2C$

$C = 3$

if $s=3$; $-4 = 4A$

$A = -1$

if $s=0$; $-4 = A + 3B - 3C$

$-4 = -1 + 3B - 3(3)$

$3B = -4 + 1 + 9$

$6 = 3B$

$B = 2$

$\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-1}{s-3} + \frac{2}{s-1} + \frac{3}{(s-1)^2}$

$= \mathcal{L}^{-1}\frac{1}{s-3} + \mathcal{L}^{-1}\frac{2}{s-1} + \mathcal{L}^{-1}\frac{3}{(s-1)^2}$

$= e^{3t} + 2e^{t} + 3te^{t}$

v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s^2+4s+16+4} = \frac{s-5}{s^2+4s+20} = \frac{s-5}{s(s+4)+20} = \frac{A}{s+20} + \frac{B}{s+4}$

$$\frac{s-5}{(s+2)(s+2)+16} = \frac{s-5}{(s+2)^2+4^2} = \frac{5}{(s+2)^2+4^2} = \frac{5}{4(s+2)^2+4}$$

$$L^{-1}\left\{\frac{5}{4(s+2)^2+4^2}\right\} = e^{-2t}\cos 4t - 5/4\, e^{-2t}\sin 4t$$