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ASSIGNMENT ON: ENG381 (NO. 4)

① $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

Soln

$$(1-x^2)y^{(2)} - 2xy^{(1)} + 2y^{(0)} = 0$$

$$(1-x^2)y^{(n+2)} + 2xny^{(n+1)} + (n^2-n)y^n + 2xy^{(n+1)} + 2ny^n + 2y^n = 0$$

$$(1-x^2)y^{(n+2)} + y^{(n+1)}(2xn+2x) + y^n(n^2+n+2) = 0$$

when $x \to 0$

$$y^{(n+2)} + y^n(n^2+n+2) = 0$$

$$y^{(n+2)} = -(n^2+n+2)y^n$$

$$y^{(n+2)} = (-n^2 - n - 2)y^n$$

when

$n=0$	$y^2 = -2y^{(0)}$
$n=1$	$y^3 = -4y^{(1)}$
$n=2$	$y^4 = -8y^{(2)} = (-8)(-2)y^{(0)}$
$n=3$	$y^5 = -14y^{(3)} = (-14)(-4)y^{(1)}$
$n=4$	$y^6 = -22y^{(4)} = (-22)(-8)(-2)y^{(0)}$
$n=5$	$y^7 = -32y^{(5)} = (-32)(-14)(-4)y^{(1)}$

$$y = y^{(0)} + xy^{(1)} + \frac{x^2}{2!}(-2)y^{(0)} + \frac{x^3}{3!}(-4)y^{(1)} + \frac{x^4}{4!}(16)y^{(0)} + \frac{x^5}{5!}(56)y^{(1)} + \frac{x^6}{6!}(-352)y^{(0)} + \frac{x^7}{7!}(-1792)y^{(1)}$$

$$y = y^{(0)}\left[1 + \frac{x^2}{2!}(-2) + \frac{x^4}{4!}(16) + \frac{x^6}{6!}(-352)\right] + y^{(1)}\left[x + \frac{x^3}{3!}(-4) + \frac{x^5}{5!}(56) + \frac{x^7}{7!}(-1792)\right]$$

$$y = y^{(0)}\left[1 + x + \frac{2x^4}{3} - \frac{22}{45}x^6\right] + y^{(1)}\left[x - \frac{2}{3}x^3 + \frac{7}{15}x^5 - \frac{16}{45}x^7\right]$$

$$y = y^{(0)}\left[\frac{45 + 45x + 30x^4 - 22x^6}{45}\right] + y^{(1)}\left[\frac{45x - 30x^3 + 21x^5 - 16x^7}{45}\right]$$

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Question 2.

(i) $\mathcal{L}\{3e^{-4t}+5e^{4t}\}$

$= \frac{3}{s+4} + \frac{5}{s-4}$ //

(ii) $\mathcal{L}\{\sin 4t\} + \mathcal{L}\{\cos 4t\}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16}$ //

(iii) t^3+2t^2-t+4

$= \mathcal{L}\{t^3\}+2\mathcal{L}\{t^2\} - \mathcal{L}\{t\} + \mathcal{L}\{4\}$

$= \frac{3!}{s^4} + \frac{2\times 2!}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $e^{-2t}\cos 5t = \mathcal{L}\{e^{-2t}\cos 5t\}$

$= \frac{s+2}{(s+2)+25}$ //

(v) $t\sin 3t = \mathcal{L}\{t\sin 3t\}$

$= -\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$

$= -\frac{-6s}{(s^2+9)^2}$

$\mathcal{L}\{t\sin 3t\} = \frac{6s}{(s^2+9)^2}$ //

(vi) $\mathcal{L}\left[\frac{e^{-t}-e^{-2t}}{t}\right]$

$\lim_{t\to 0} e^0 - e^0 = 0$

$\mathcal{L}\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \int_s^\infty \mathcal{L}[e^{-t}]\,ds - \int_s^\infty \mathcal{L}[e^{-2t}]\,ds$

$= \int_s^\infty \frac{1}{s+1} - \int_s^\infty \frac{1}{s+2}$

$= \left(\ln\left[\frac{s+1}{s+2}\right]\right)\Big|_s^\infty$

$= \left[\ln\left(\frac{\infty+1}{\infty+2}\right) - \ln\frac{s+1}{s+2}\right]$

$= \left[\ln\frac{\infty}{\infty} - \ln\frac{s+1}{s+2}\right]$

$= \left[\ln\infty - \ln\frac{s+1}{s+2}\right]$

$\mathcal{L}\left[\frac{e^{-t}-e^{-2t}}{t}\right] = \ln\left[\frac{\infty(s+2)}{s+1}\right]$

(vii) $\mathcal{L}[e^{4t}\cos 2t]$

$= \frac{s-4}{(s-4)+4}$

(viii) $t\sin 2t$

$\mathcal{L}[t\sin 2t] = -\frac{d}{ds}\left(\frac{2}{s^2+4}\right)$

$= -\left(\frac{-4s}{[s^2+4]^2}\right)$

$\mathcal{L}[t\sin 2t] = \frac{4s}{(s^2+4)^2}$

(ix) t^3+4t^2+5

$\mathcal{L}[t^3+4t^2+5] = \mathcal{L}[t^3] + \mathcal{L}[4t^2] + \mathcal{L}[5]$

$= \frac{3!}{s^4} + \frac{4\times 2!}{s^3} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $e^{3t}(t^2+4)$

$\mathcal{L}\{e^{3t}(t^2+4)\} = \mathcal{L}\{e^{3t}(t^2)\} + \mathcal{L}\{e^{3t}(4)\}$

$= \frac{2!}{(s-3)^3} + \frac{4}{s-3}$

$\therefore \mathcal{L}\{e^{3t}(t^2+4)\} = \frac{2}{(s-3)^3} + \frac{4}{s-3}$

Question 2 (cont.)

(xi) $t^2 \cos t$

$$\mathcal{L}\{t^2\cos t\} = \frac{-d}{ds}\left(\frac{-d}{ds}\right)\left[\frac{s}{s^2+1}\right]$$

$$= \frac{-d}{ds}\left[\frac{s^2-1}{(s^2+1)^2}\right]$$

$$= \frac{(s^2+1)^2(2s) - (s^2-1)(4s(s^2+1))}{(s^2+1)^2}$$

$$= \frac{2s(s^2+1)^2 - 4s(s^2+1)(s^2-1)}{}$$

$$= \frac{\cancel{2s(s^2+1)}}{(s^2+1)^2}$$

$$= \frac{2s\cancel{(s^2+1)}\left[(s^2+1) - 2s(s^2-1)\right]}{(s^2+1)^4}$$

$$\mathcal{L}\{t^2\cos t\} = \frac{2s\left[(s^2+1) - 2s(s^2-1)\right]}{(s^2+1)}$$

$$\mathcal{L}\{t^2\cos t\} = \left(\frac{2s(s^2+1-2s^3+2s)}{s^2+1}\right)$$

$$= \left(\frac{2s^3 - 2s - 4s^4 + 4s^2}{s^2+1}\right)$$

$$\mathcal{L}\{t^2\cos t\} = \frac{-2s^3+2s+4s^4-4s^2}{s^2+1}$$

$$\mathcal{L}\{t^2\cos t\} = \frac{4s^4-2s^3-4s^2+2s}{s^2+1}$$

(xii) $\frac{\sinh 2t}{t}$

$$\mathcal{L}\left\{\frac{\sinh 2t}{t}\right\} = \int_s^\infty \mathcal{L}\{\sinh 2t\}\,ds$$

$$= \int_s^\infty \frac{2}{s^2-2^2}\,ds$$

$$= 2\int_s^\infty \frac{1}{s^2-2^2}\,ds$$

$$= 2\left[\frac{-1}{2}\tan^{-1}\frac{s}{2}\right]_s^\infty$$

$$= -\tan^{-1}\frac{s}{2}\Big|_s^\infty$$

$$= -\tan^{-1}\frac{\infty}{2} + \tan^{-1}\frac{s}{2}$$

$$= \tan^{-1}\frac{s}{2} - \tan^{-1}\frac{\infty}{2}$$

$$= \tan^{-1}\frac{s}{2} - 90^\circ$$

$$= \tan^{-1}\frac{s}{2} - \frac{\pi}{2}$$

$$= \cancel{\tan^{-1}\frac{s}{2}} - \tan^{-1}\frac{a}{s} - \cancel{\tan^{-1}\frac{s}{2}}$$

$$\mathcal{L}\left\{\frac{\sinh 2t}{t}\right\} = -\tan^{-1}\frac{2}{s}$$

$$\mathcal{L}\left\{\frac{\sinh 2t}{t}\right\} = \tan^{-1}\frac{s}{2}$$

Question 3

(i) $\frac{s-5}{(s-3)(s-4)}$

$$F(t) = \mathcal{L}^{-1}\left\{\frac{s-5}{(s-3)(s-4)}\right\}$$

But

$$\frac{s-5}{(s-3)(s-4)} = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$s-5 = A(s-4) + B(s-3)$$

when $s \to 3$

$$-2 = -A$$

$$A = 2$$

when $s \to 4$

$$-1 = B$$

$$\therefore f(t) = \mathcal{L}^{-1}\left\{\frac{2}{s-3}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\}$$

$f(t) = 2e^{3t} - e^{4t}$

(ii) $\dfrac{2s-6}{(s-2)(s-4)}$

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{2s-6}{(s-2)(s-4)}\right\}$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{s-2} + \dfrac{B}{s-4}$

$2s-6 = A(s-4) + B(s-2)$

when $s=2$

$-2 = -2A$

$A = 1$

when $s=4$

$2 = 2B$

$B = 1$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{1}{(s-2)} + \dfrac{1}{(s-4)}$

$\mathcal{L}^{-1}\left\{\dfrac{2s-6}{(s-2)(s-4)}\right\} = \mathcal{L}^{-1}\left\{\dfrac{1}{s-2}\right\} + \mathcal{L}^{-1}\left\{\dfrac{1}{s-4}\right\}$

$f(t) = e^{2t} + e^{4t}$

(iii) $\dfrac{5s-8}{s(s-4)} = \dfrac{A}{s} + \dfrac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

when $s=0$

$-8 = -4A$

$A = 2$

when $s=4$

$12 = 4B$

$B = 3$

$\dfrac{5s-8}{s(s-4)} = \dfrac{2}{s} + \dfrac{3}{s-4}$

$\mathcal{L}^{-1}\left\{\dfrac{5s-8}{s(s-4)}\right\} = f(t)$

$f(t) = \mathcal{L}^{-1}\left\{\dfrac{2}{s}\right\} + \mathcal{L}^{-1}\left\{\dfrac{3}{s-4}\right\}$

$f(t) = 2 + 3e^{4t}$

(iv) $\dfrac{s^2-3s-4}{(s-3)(s-1)^2} = \dfrac{A}{s-3} + \dfrac{B}{(s+1)} + \dfrac{C}{(s-1)^2}$

$= \dfrac{A(s-1)^2 + B(s-3)(s-1) + C(s-3)}{(s-3)(s-1)^2}$

$s^2-3s-4 = A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$= As^2 - 2As + A + Bs^2 - 4Bs + 3B + Cs - 3C$

$= s^2(A-B) + s(-2A-4B+C) + A+3B-3C$

Comparing coefficient

$1 = A+B$ — ①

$-3 = -2A-4B+C$

$3 = 2A+4B-C$ — ②

$-4 = A+3B-3C$ — ③

from equ ①

$A = B-1$

substituting A in equ ①

$1 = B-1+B$

$1 = 2B-1$

$2B = 2$

$B = 1$

still substituting A in equ ②

$3 = 2(B-1) + 4B - C$

$3 = 2B-2+4B-C$

$3+2 = 6B-C$

$5 = 6B-C$

Question 3 (Contd.)

(iv) contd.

$5 = 6B - C$

Also $B = 1$

$5 = 6 - C$

$C = 1$

Putting the values of B and C in equation (5)

$-4 = A + 3(1) - 3(1)$

$-4 = A + \cancel{3} - \cancel{3}$

$A = -4$

$$\frac{s^2 - 3s - 4}{(s-3)(s-1)^2} = \frac{-4}{s-3} + \frac{1}{(s-1)} + \frac{1}{(s-1)^2}$$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{s^2-3s-4}{(s-3)(s-1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{-4}{s-3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\right\}$$

$$f(t) = -4e^{3t} + e^{t} + te^{2t}$$

$$f(t) = \underline{te^{2t} + e^{t} - 4e^{3t}}$$

(v) $\dfrac{s-5}{s^2+4s+20}$

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